Performing Operations on Rational Expressions

Part Three: Sums and Differences

Learning Targets

I can:

- determine, in simplified form, the sum of rational expressions (**AN5.3**)
- determine, in simplified form, the difference of rational expressions (**AN5.4**)

Adding and Subtracting Rational Expressions with the Same Denominator

Let a, b and c be polynomials where $c \neq 0$.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \text{and} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Strategy for Adding and Subtracting Rational Expressions with the Same Denominator

1. State all non-permissible values.

2. Add or subtract numerators as indicated.

3. Simplify.

Sample Problems

Simplify.

a) $\dfrac{5}{3x} + \dfrac{7}{3x}$

b) $\dfrac{3x}{x-1} - \dfrac{x+5}{x-1}$

c) $\dfrac{4x+1}{2x-1} - \dfrac{2x-3}{2x-1}$

Determining the Least Common Denominator

You can add or subtract rational expressions only when you have a common denominator.   The least common denominator (LCD) of two or more rational expressions is the smallest expression that is divisible by each of the given denominators. (This is equivalent to saying that the least common denominator is the least common multiple (LCM) of all denominators in the collection.)

Sample Problems

Find the lowest common denominator (LCD) of the rational expressions.

Monomial Denominators

a) $\dfrac{1}{24x}, \ \dfrac{x+2}{6x^3}$

b) $\dfrac{b}{-8a^5}, \ \dfrac{c}{4a^3}$

c) $\dfrac{4}{15x^2}, \ \dfrac{x^2-1}{20x^3}$

d) $\dfrac{3}{14x^3y^2}, \ \dfrac{x+4}{21xy}$

Polynomial Denominators

a) $\dfrac{5}{x-2}, \ \dfrac{3}{x-4}$

b) $\dfrac{-7x}{x-6}, \ \dfrac{y}{x+5}$

c) $\dfrac{x+3}{x-7}, \ \dfrac{x+1}{x-1}$

Note: To determine the LCD for the following pairs of rational expressions, factor each denominator and determine the maximum number of times each factor occurs in any factorization.

d) $\dfrac{x-5}{x^2-9}, \ \dfrac{x-4}{x-3}$

e) $\dfrac{2}{x^2+4x-5}, \ \dfrac{x^2+2}{x^2+7x+10}$

f) $\dfrac{6}{(x+2)^2}, \ \dfrac{3x+4}{x^2-x-6}$

Adding and Subtracting Rational Expressions with Different Denominators

Strategy

1. Completely factor all polynomial denominators.

2. Determine the least common denominator (LCD).

   Note: Use the LCD to identify the non-permissible values.

3. Find the new numerator for each fraction. To do this, compare the denominator of each of the original fractions to the LCD and then multiply each numerator by whatever factor(s) are missing.

4. Add or subtract the numerators over the LCD. When subtracting, notice the subtraction sign is moved into the numerator so it can be distributed later if need be.

5. Simplify the numerator by distributing and combining like terms.

6. Check to make sure your answer is in written in simplest form.

Sample Problems

a) $\dfrac{9}{8x^2} + \dfrac{5}{12x^3}$

b) $\dfrac{3}{2x} - \dfrac{7}{5x^2y}$

c) $\dfrac{10}{3x} - \dfrac{x}{x+2}$

d) $\dfrac{y}{y+1} + \dfrac{3}{y+2}$

e) $\dfrac{x+4}{x^2+3x-10} - \dfrac{x-1}{x^2+2x-8}$

f) $\dfrac{2a-1}{a^2+2a-8} - \dfrac{a-1}{a^2-4}$

g) $\dfrac{k+7}{k^2+6k+9} + \dfrac{k-5}{k^2-5k-24}$

h) $\dfrac{x+2}{2x^2-x-15} - \dfrac{x-2}{x^2+2x-15}$