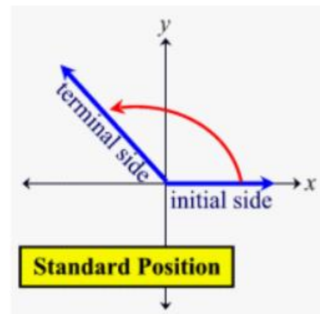
Types of Angles

(Angles in Standard Position, Quadrantal Angles, Principal Angle and Reference Angles)

Standard Position

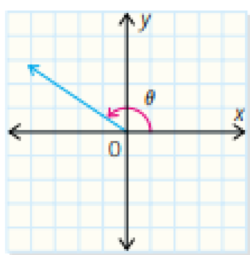


An angle is in standard position if its vertex is located at the origin and one ray is on the positive x-axis. The ray on the x-axis is called the **initial side/arm** and the other ray is called the **terminal side/arm**.

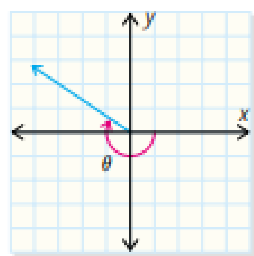
If the terminal side of an angle lies "on" the axes (ie. $0°, 90°, 180°, 270°, 360°, 450°$, etc.) the angle is called a **quadrantal angle**.

Angles in standard position may be positive or negative.

- If measured in a **counterclockwise direction** the measurement is **positive**.
- If measured in a **clockwise** direction the measurement is **negative.**

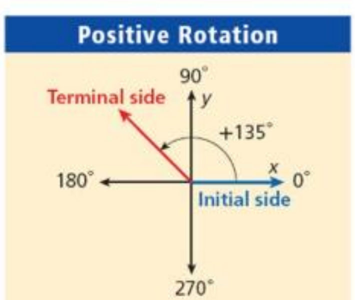


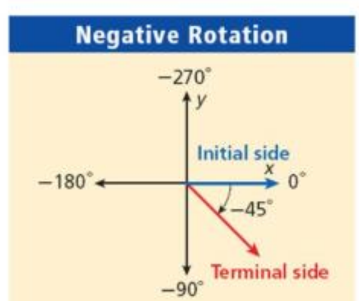
a positive angle



a negative angle

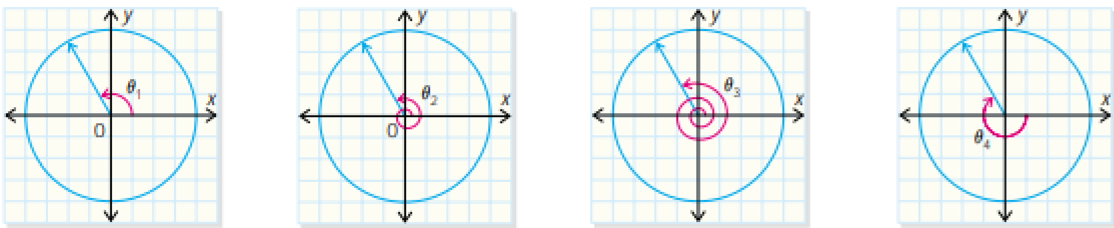
Examples





Coterminal and Principal Angles

Coterminal angles are angles in standard position that share the same terminal arm.



$120°$ $480°$ $840°$ $-240°$

The **principal angle** is the smallest positive angle in a set of coterminal angles ($0° < \theta < 360°$). The principal angle of the set of coterminal angles given above is 120°.

By adding or subtracting multiples of one full rotation, you can determine both positive and negative angles that are coterminal with a given angle.

Examples

Write two positive and two negative angles coterminal with each angle given below.

a) 40°

40° + 360° = 400° 40° - 360° = -320°

400° + 360° = 760° -320° - 360°= -680°

b) $\frac{2\pi}{3}$

$$\frac{2\pi}{3} + 2\pi = \frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3} \qquad \frac{2\pi}{3} - 2\pi = \frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{4\pi}{3}$$

$$\frac{8\pi}{3} + 2\pi = \frac{8\pi}{3} + \frac{6\pi}{3} = \frac{14\pi}{3} \qquad -\frac{4\pi}{3} - 2\pi = -\frac{4\pi}{3} - \frac{6\pi}{3} = -\frac{10\pi}{3}$$

General Form for Expressing All Coterminal Angles

Any given angle has an infinite number of angles coterminal with it, since each time you make one full rotation (positive or negative) from the terminal arm, you arrive back at the same terminal arm. Angles coterminal with an angle, θ, can be given using the expressions below.

$$\theta + 360°n,\ n \in I \qquad \text{or} \qquad \theta + 2\pi n,\ n \epsilon I$$

Note: n represents the number (and direction) of rotations. For example, $n = 3$ represents three full rotations in the counterclockwise (positive) direction and $n = -2$ represents two full rotations in the clockwise (negative) direction.

Examples

Write an expression for all of the angles coterminal with each angle given below.

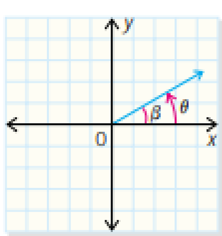
a) 40° $\qquad$ 40° + $360°n, n \in I$

b) $\frac{2\pi}{3}$ $\qquad$ $\frac{2\pi}{3} + 2\pi n,\ n \in I$

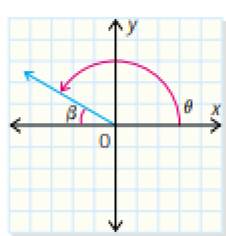
Reference Angles

Associated with every angle drawn in standard position (except quadrantal angles) there is another angle called the **reference angle**, θ_r. The reference angle is the **acute angle** (the smallest angle) formed by the terminal side of the given angle and the x-axis. Reference angles may appear in all four quadrants. Angles in quadrant I are their own reference angles.

Note: In the diagrams below the reference angle is represented by β.

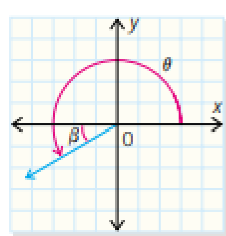


$\beta = \theta$



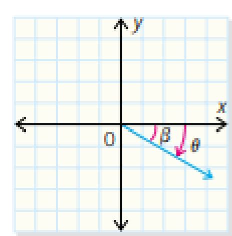
$\beta = 180° - \theta$

$\beta = \pi - \theta$



$\beta = \theta - 180°$

$\beta = \theta - \pi$



$\beta = 360° - \theta$

$\beta = 2\pi r - \theta$