

Change of Base Formula

For some purposes we find it useful to change from logarithms in one base to logarithms in another base.

Suppose we are given $\log_a x$ and want to find $\log_b x$.

Let

$$y = \log_b x$$

We write this in exponential form and take the logarithm, with base a , of each side.

$$b^y = x \quad \text{Exponential form}$$

$$\log_a(b^y) = \log_a x \quad \text{Take } \log_a \text{ of each side}$$

$$y \log_a b = \log_a x \quad \text{Law 3}$$

$$y = \frac{\log_a x}{\log_a b} \quad \text{Divide by } \log_a b$$

This proves the following formula.

CHANGE OF BASE FORMULA

$$\log_a x = \frac{\log_a x}{\log_a b}$$

Use the Change of Base Formula and common or natural logarithms to evaluate each logarithm, correct to five decimal places.

(a) $\log_8 5$ (b) $\log_9 20$

Solution:

(a) We use the Change of Base Formula with $b = 8$ and $a = 10$:

$$\log_8 5 = \frac{\log_{10} 5}{\log_{10} 8} \approx 0.77398$$

(b) We use the Change of Base Formula with $b = 9$ and $a = e$:

$$\log_9 20 = \frac{\ln 20}{\ln 9} \approx 1.36342$$