Calculus 120

Check Your Understanding – Curve Sketching and Optimization Problems

Answer on your own paper.

1. Determine the exact values of the critical numbers for the function given below.

$$f(x) = \sqrt[3]{x^3 - 9x}$$

2. Find the absolute maximum value and absolute minimum value for the given interval.

$$f(x) = 6x^3 - 6x^4 + 5, \quad -1 \le x \le 2$$

3. Bubba Newton was asked to find intervals of increase and decrease for the curve $y = x^{\frac{1}{3}}(x^2 - 7)$. With the help of his brother, Figgy, Bubba determined the first derivative of the function to be $y' = \frac{7}{3}x^{-\frac{2}{3}}(x^2 - 1)$. Neither Bubba nor Figgy know what do next. Help the poor boys out… complete the solution. *Include a neatly labeled number line as part of your solution.*

4. Determine whether the MVT can be applied to *f* on the closed interval. If the MVT can be applied, find all values of *c* given by the theorem. If the MVT cannot be applied, explain why not.

   a) $f(x) = x^3 + 24x - 16$, [0, 4]

   b) $h(x) = 4 - |x - 3|$, [-2, 5]

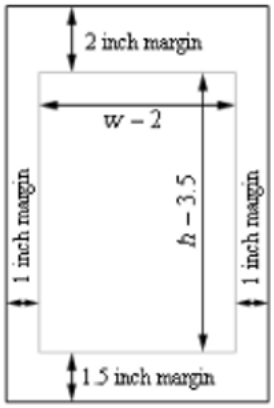
5. Determine whether Rolle's Theorem can be applied to *f* on the closed interval. If Rolle's Theorem can be applied, find all values *c* in the open interval such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

   $f(x) = x(1 - x)$, [-1, 1]

6. Determine the intervals of concavity on the graph of the function $g(x)$ given its second derivative below. *Include a neatly labeled number line as part of your solution.*

$$g''(x) = \frac{x+1}{x^{\frac{2}{3}}(x+3)^{\frac{1}{3}}}$$

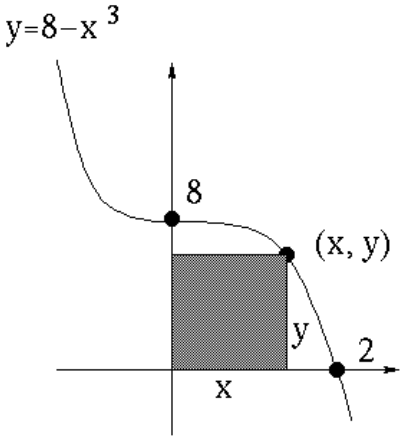
7. A printer needs to make a poster that will have a total area of 200 in^2 and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom. What dimensions of the poster will give the largest printed area?



8. Determine the points $y = x^2 + 1$ that are closest to the point (0, 2).

Challenge Problem

9. Determine the dimensions of the rectangle of largest area that can be inscribed inside the closed region bounded by the x-axis, y-axis and the graph of $y = 8 - x^2$.



10. Consider the function $f(x) = \frac{x^2 + 5x + 4}{x^2}$ and its first and second derivatives shown below.

$$f'(x) = \frac{-(5x+8)}{x^3} \qquad f''(x) = \frac{10x + 24}{x^4}$$

Complete the table and draw a neatly labeled graph of *f*(*x*) on a sheet of graph paper.

It would be a good idea to neatly label and show your work on loose leaf.

Domain	
y-intercept(s)	
x-intercept(s)	
Symmetry	
VA(s)	
HA(s)	
Interval(s) of Inc.	
Interval(s) of Dec.	
Relative Maximum Value(s)	
Relative Minimum Value(s)	
Concave Up	
Concave Down	
Inflection Point(s)	