

Sample Problem #1

$$f(x) = x^3 - 3x^2$$

Domain

$$(-\infty, \infty)$$

Intercepts

$$x\text{-int, let } y = 0$$

$$0 = x^3 - 3x^2$$

$$0 = x^2(x - 3)$$

$$x = 0, x = 3$$

$$(0, 0), (3, 0)$$

$$y\text{-int, let } x = 0$$

$$y = 0^3 - 3(0)^2$$

$$y = 0$$

$$(0, 0)$$

Symmetry

$$f(-x) = (-x)^3 - 3(x)^2 = -x^3 - 3x^2$$

$$f(-x) \neq f(x)$$

$$f(-x) \neq -f(x)$$

Neither even nor odd

Asymptotes

$f(x)$ is a polynomial

∴ There are no horizontal or vertical asymptotes

Intervals of Increase and Decrease

$$f'(x) = 0$$

$f(x)$ is undefined

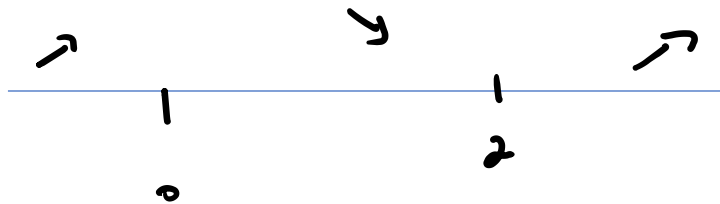
$$f(x) = x^3 - 3x^2$$

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$$f'(x) = 3x^2 - 6x$$

$$0 = 3x(x-2)$$

$$x = 0, x = 2$$



Test Values	$f'(x)$	
$x = -1$	+	Inc
$x = 0.5$	-	Dec
$x = 3$	+	Inc

Intervals of Inc

$$(-\infty, 0) \cup (2, \infty)$$

Interval of Dec.

$$(0, 2)$$

Local Max : $f(0) = 0$

Local Min : $f(2) = -4$

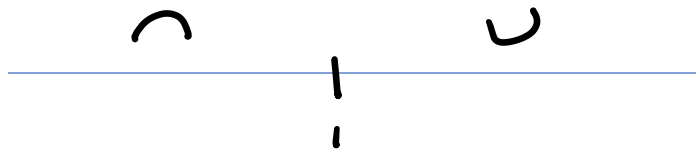
Intervals of Concavity and
Inflection Points

$$f(x) = x^3 - 3x^2$$

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6 = 6(x - 1)$$

$f''(x) = 0$	$f''(x)$ is und.
$6(x - 1) = 0$	-
$x = 1$	



Test Values	$f''(x)$	
$x = 0$	-	down
$x = 2$	+	up

Concave Up $(1, \infty)$

Concave Down $(-\infty, 1)$

Point of Inflection

$$f(1) = 1^3 - 3(1)^2 = 1 - 3 = -2$$

$(1, -2)$

Domain	$(-\infty, \infty)$
y-intercept(s)	$(0, 0)$
x-intercept(s)	$(0, 0), (3, 0)$
Symmetry	neither
VA(s)	-
HA(s)	-
Interval(s) of Inc.	$(-\infty, 0) \cup (2, \infty)$
Interval(s) of Dec.	$(0, 2)$
Relative Maximum Value(s)	$f(0) = 0$
Relative Minimum Value(s)	$f(2) = 4$
Concave Up	$(1, \infty)$
Concave Down	$(-\infty, 1)$
Inflection Point(s)	$(1, -2)$

