Sample Problem #2

$f(x) = \frac{x^2}{1-x^2}$, $f'(x) = \frac{2x}{(1-x^2)^2}$, $f''(x) = \frac{8x^2 - 2x + 2}{(1-x^2)^3}$

Domain

Inputs : $x = 1$, $x = -1$ $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Intercepts

x-int , let $y = 0$

$$0 = \frac{x^2}{1-x^2}$$

$$0 = x^2$$

$$0 = x$$ $(0, 0)$

y-int , let $x = 0$

$$f(0) = \frac{0^2}{1-0^2} = 0$$

$(0, 0)$

Symmetry

$f(-x) = \frac{(-x)^2}{1-(-x)^2} = \frac{x^2}{1-x^2}$, $f(-x) = f(x)$

even

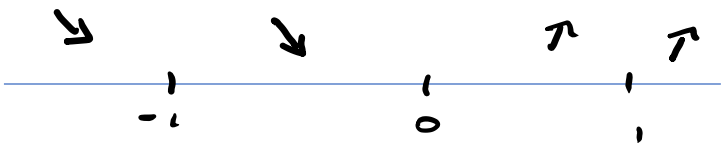
Asymptotes

VA $x = 1, \quad x = -1$

HA $y = -1$

Intervals of Increase and Decrease

$f'(x) = 0$	$f(x)$ is undefined
$\frac{2x}{(1-x^2)^2} = 0$	$(1-x^2)^2 = 0$
$2x = 0$	$1 - x^2 = 0$
$x = 0$	$1 = x^2$
	$x = \pm 1$



Test Values	$f'(x)$	
$x = -2$	$-$	dec
$x = -0.5$	$-$	dec
$x = 0.5$	$+$	inc
$x = 2$	$+$	inc

Intervals of Inc

$(0, 1) \cup (1, \infty)$

Intervals of Dec.

$(-\infty, -1) \cup (-1, 0)$

Local Max : —

Local Min : $f(0) = 0$

Intervals of Concavity and Inflection Points

$$f''(x) = \frac{8x^2 - 2x + 2}{(1-x^2)^3}$$

$f''(x) = 0$	$f''(x)$ is und.
$0 = 8x^2 - 2x + 2$	$(1-x^2)^3 = 0$
$0 = 2(4x^2 - x + 1)$	$1 - x^2 = 0$
No solution!	$1 = x^2$
	$\pm 1 = x$

Test Values	$f''(x)$
$x = -2$	$-$
$x = 0$	$+$
$x = 2$	$-$

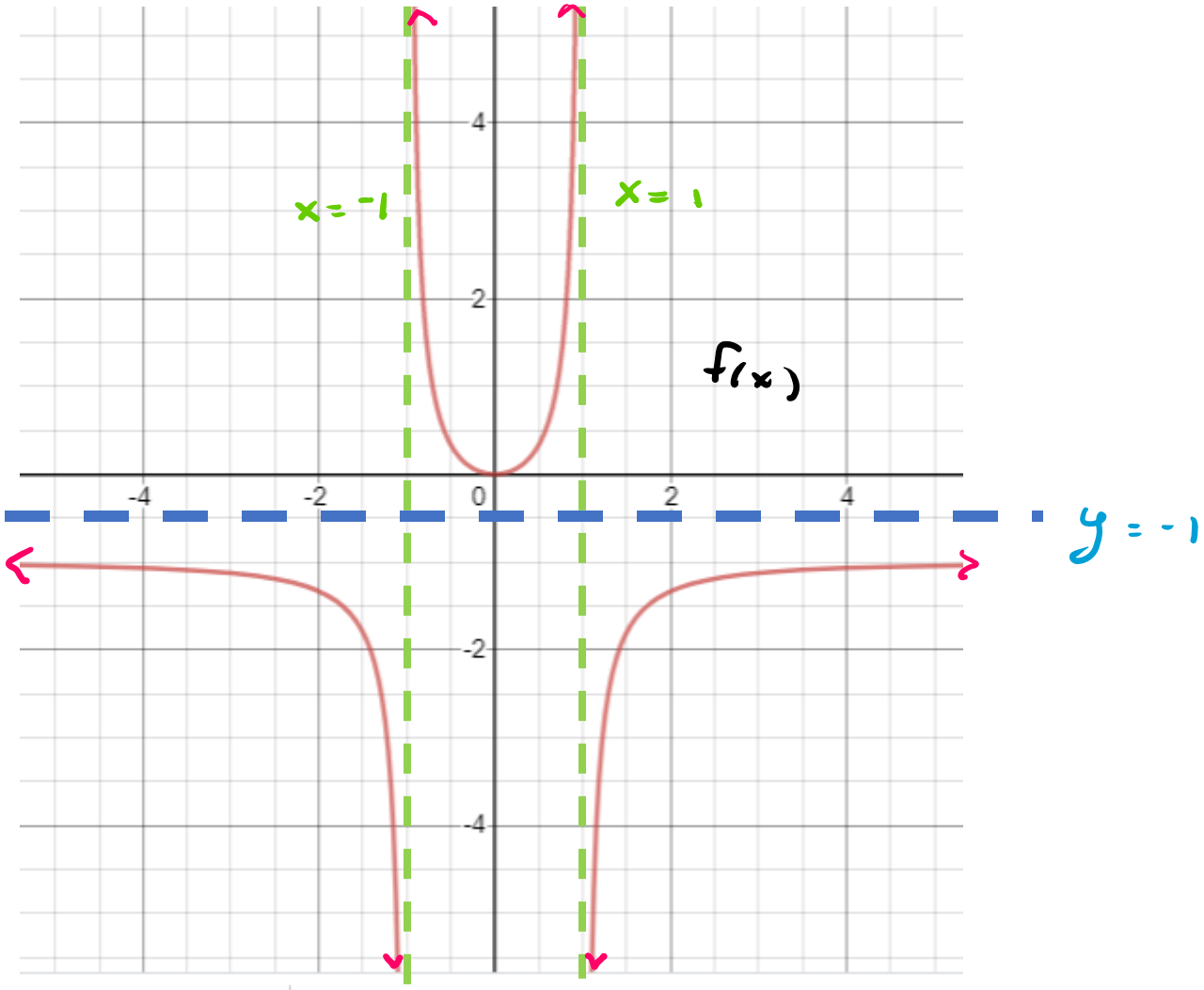
Concave Up $(-1, 1)$

Concave Down $(-\infty, -1) \cup (1, \infty)$

Point of Inflection

none !

Domain	$(-\infty, 1) \cup (-1, 1) \cup (1, \infty)$
y-intercept(s)	$(0, 0)$
x-intercept(s)	$(0, 0)$
Symmetry	even
VA(s)	$x = 1, \quad x = -1$
HA(s)	$y = -1$
Interval(s) of Inc.	$(0, 1) \cup (1, \infty)$
Interval(s) of Dec.	$(-\infty, -1) \cup (-1, 0)$
Relative Maximum Value(s)	—
Relative Minimum Value(s)	$f(0) = 0$
Concave Up	$(-1, 1)$
Concave Down	$(-\infty, -1) \cup (1, \infty)$
Inflection Point(s)	—


x = -1
x = 1
f(x)
y = -1
4
2
-2
-4
-4
-2
0
2
4