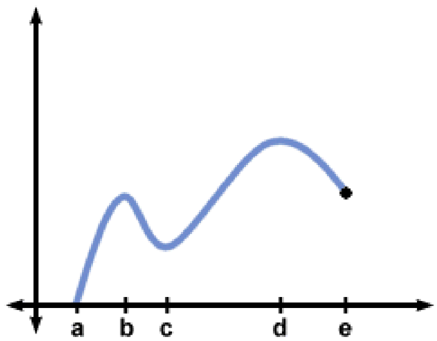
Extreme Values



On the graph above of the function *f* on the closed interval [*a*, *e*], the point (*a*, *f* (*a*)) represents the absolute minimum, and the point (*d*, *f* (*d*)) represents the absolute maximum.

To define these terms more formally: a function *f* has an absolute maximum at $x = b$ if $f(b) \geq f(x)$ for all *x* in the domain of *f*. A function *f* has an absolute minimum at $x = b$ if $f(b) \leq f(x)$ for all *x* in the domain of *f*.

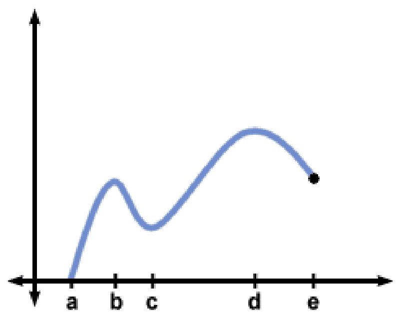
Together, the absolute minimum and the absolute maximum are known as the **absolute extrema** of the function.

Local Extrema

A function *f* has a local maximum at *x* = a if *f* (a) is the largest value that *f* attains "near a." Similarly, a function *f* has a local minimum at *x* = a if *f* (a) is the smallest value that *f* attains "near a."

To define these terms more formally: a function $f(x)$ has a local (or relative) maximum at $x = b$ if there is an open interval *I* in which $f(b) \geq f(x)$ for all *x* in *I*. A function $f(x)$ has a local (or relative) minimum at $x = b$ if there is an open interval *I* in which $f(b) \leq f(x)$ for all *x* in *I*.

In the graph below, f has a local maximum at $x = b$ and at $x = d$, and has a local minimum at $x = c$.



Taken together, the local maxima and local minima are known as the **local extrema**. A local minimum or local maximum may also be called a **relative minimum** or **relative maximum**.

Problem: Do absolute extrema always count as local extrema?

No. Absolute extrema that occur at the endpoints of an interval are not considered local extrema because the definition of a local extremum requires that there be an open interval I containing the extremum, but if the point is an endpoint, no such open interval exists.