Exponential Growth or Exponential Decay

If we are given an exponential function and asked to predict if the resulting graph would be exponential growth or exponential decay, how can we correctly answer the question without actually drawing the graph? The key to correctly answering the question is to look at the base of the exponential function. Consider the following exponential functions and try to predict growth or decay by looking at the base of the function:

$$f(x)=\left(\frac{4}{3}\right)^x \text{ and } f(x)=\left(\frac{6}{5}\right)^x$$

The function with the base of 4/3 will be exponential growth and the other function with a base of 6/5 will also be exponential growth. The key to determining growth or decay depends on if the base, b, is less than one or greater than one. If the base is greater than one, $b > 1$, we will get growth, an increase as the graph goes from left to right. If the base is less than one, $0 < b < 1$, we will get decay, a decrease as the graph goes from left to right. In the two functions used above, both 4/3 and 6/5 are greater than 1 which is why both graphs would result in exponential growth.

Summary

Here is a summary of the features of the graph of an exponential function, $f(x) = b^x$.

Features of the Graph of Exponential Functions in the Form $f(x) = b^x$ or $y = b^x$

- The domain of $f(x) = b^x$ is all real numbers.
- The range of $f(x) = b^x$ is all positive real numbers, $f(x) > 0$ or $y > 0$.
- The graph of $f(x) = b^x$ must pass through the point (0, 1) because any number, except zero, raised to the zero power is 1. The y-intercept of the graph $f(x) = b^x$ is always 1.
- The graph of $f(x) = b^x$ always has a horizontal asymptote at the x-axis ($f(x) = 0$ or $y = 0$) because the graph will get closer and closer to the x-axis but never touch the x-axis.
- If $0< b < 1$ the graph of $f(x) = b^x$ will decrease from left to right and is called exponential decay.
- If $b > 1$ the graph of $f(x) = b^x$ will increase from left to right and is called exponential growth.