

Laws of Logarithms

Since logarithms are exponents, the Laws of Exponents give rise to the Laws of Logarithms.

LAWS OF LOGARITHMS

Let a be a positive number, with $a \neq 1$. Let A , B , and C be any real numbers with $A > 0$ and $B > 0$.

Law	Description
1. $\log_a(AB) = \log_a A + \log_a B$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
2. $\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$	The logarithm of a quotient of numbers is the difference of the logarithms of the numbers.
3. $\log_a(A^C) = C \log_a A$	The logarithm of a power of a number is the exponent times the logarithm of the number.

Evaluate each expression.

(a) $\log_4 2 + \log_4 32$

(b) $\log_2 80 - \log_2 5$

(c) $-\frac{1}{3} \log 8$

Solution:

(a) $\log_4 2 + \log_4 32 = \log_4(2 \cdot 32)$ Law 1
 $= \log_4 64 = 3$ Because $64 = 4^3$

(b) $\log_2 80 - \log_2 5 = \log_2\left(\frac{80}{5}\right)$ Law 2
 $= \log_2 16 = 4$ Because $16 = 2^4$

(c) $-\frac{1}{3} \log 8 = \log(8^{-1/3})$ Law 3
 $= \log\left(\frac{1}{2}\right)$ Property of negative exponents
 ≈ -0.301 Calculator

Expanding and Combining Logarithmic Expressions

The Laws of Logarithms allow us to write the logarithm of a product or a quotient as the sum or difference of logarithms.

This process, called *expanding* a logarithmic expression, is illustrated in the next example.

Use the Laws of Logarithms to expand each expression.

(a) $\log_2(6x)$ (b) $\log_5(x^3y^6)$ (c) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right)$

Solution:

(a) $\log_2(6x) = \log_2 6 + \log_2 x$ Law 1

(b) $\log_5(x^3y^6) = \log_5 x^3 + \log_5 y^6$ Law 1
 $= 3 \log_5 x + 6 \log_5 y$ Law 3

(c) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right) = \ln(ab) - \ln \sqrt[3]{c}$ Law 2
 $= \ln a + \ln b - \ln c^{1/3}$ Law 1
 $= \ln a + \ln b - \frac{1}{3} \ln c$ Law 3

The Laws of Logarithms also allow us to reverse the process of expanding that was done in Example 2.

That is, we can write sums and differences of logarithms as a single logarithm.

This process, called *combining* logarithmic expressions, is illustrated in the next example.

Combine $3 \log x + \frac{1}{2} \log(x+1)$ into a single logarithm.

Solution:
 $3 \log x + \frac{1}{2} \log(x+1) = \log x^3 + \log(x+1)^{1/2}$ Law 3
 $= \log(x^3(x+1)^{1/2})$ Law 1