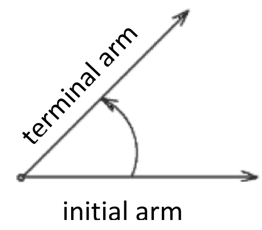
Notes – Angles and Ratios

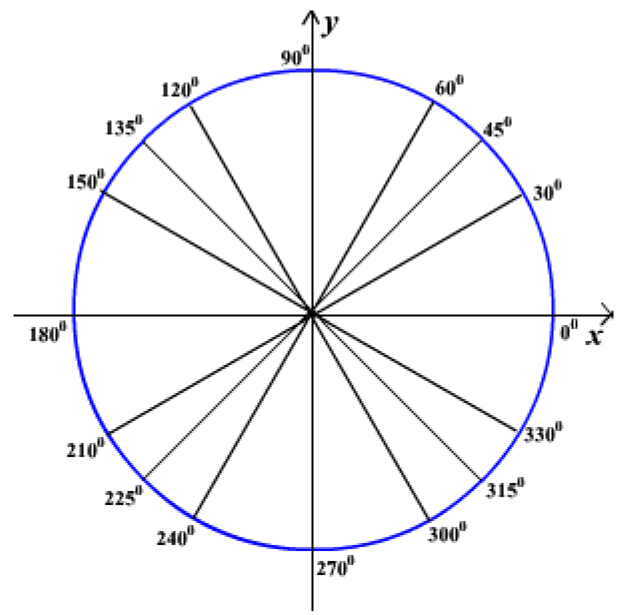


The measure of an angle is determined by the amount of rotation from the initial arm to the terminal arm.

We will be expressing the measure (or size) of an angle in degrees and radians.

Degree Measure

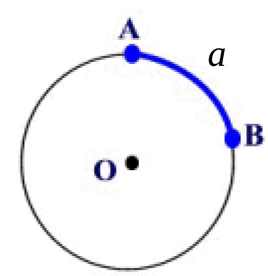
A measure of one degree, 1°, is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution.



To measure angles, it is convenient to mark degrees on the circumference of a circle. Thus a complete revolution is 360° , half a revolution is 180° , a quarter of a revolution is 90°, and so forth.

Radian Measure

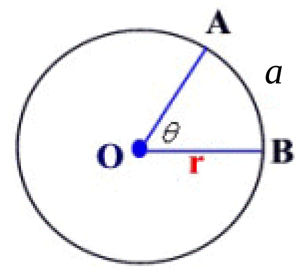
An **arc** of a circle is a "portion" of the circumference of the circle.



The **length of an arc** is simply the length of its "portion" of the circumference.
(The circumference itself can be considered an arc length.)

The length of an arc (or arc length) is traditionally symbolized by a.
In the diagram at the right, it can be said that "arc AB *subtends* angle θ".

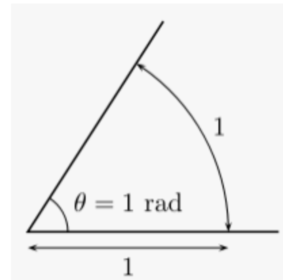
Definition: **subtend** - to be opposite to



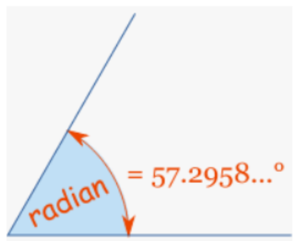
If the length of an arc, a, of a circle (think of straightening it out), is the same as the length of the circle's radius, r, a specific situation occurs. The measure of the angle, θ, created by this situation is called a **radian.**

Formal Definition of Radian

One radian is the measure of the central angle whose arc length is equal to the radius of the circle.



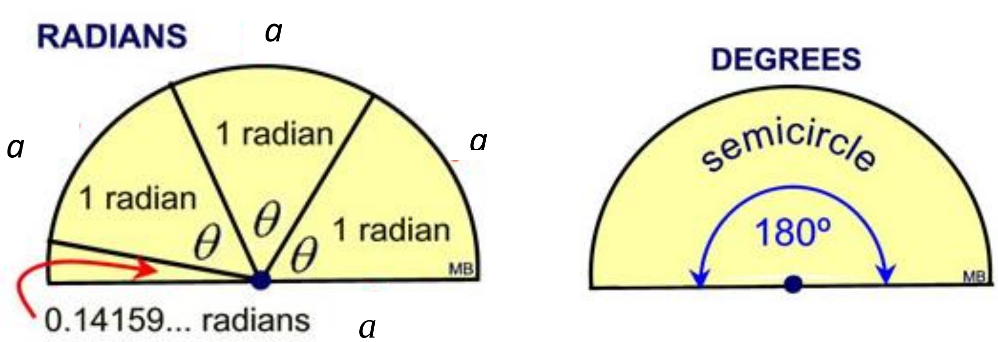
When comparing radians to degrees, one radian is approximately 57.3°.



Relationship between Radians and Degrees

Take a look at the two semicircles below.

If we "wrap" the radius, a, along the circumference of a semicircle (first diagram), we will need 3 radii plus a "bit more" to complete the semicircle. The number of radians needed to represent the central angle of a semicircle is approximately 3.14159 radians. Yes, that is π!



$$\underbrace{3.14159...}_{\pi} \text{ radians} = 180^\circ$$

$$\pi \text{ radians} = 180^\circ$$

Converting Between Angle Measures

Conversion Factors: π radians = 180°
1 radian ≈ 57.29577951° (divide by π)
2π radians = 360°

To change
from degrees to radians,
multiply by $\frac{\pi}{180^\circ}$

To change
from radians to degrees,
multiply by $\frac{180^\circ}{\pi}$

Examples

a) Convert 30° to radians. *Express your answer as an exact value in terms of* π.

$$30^\circ = 30^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{6}$$

b) Convert 65° to radians. *Express your answer as an approximate value rounded to the nearest thousandth of a radian.*

$$65^\circ = 65^\circ \times \frac{\pi}{180^\circ} = 1.113 \text{ radians}$$

c) Convert 1.75 radians to degrees. *Express your answer as an approximate value rounded to the nearest tenth of a degree.*

$$1.75 \text{ radians} = 1.75 \times \frac{180^\circ}{\pi} = 100.3^\circ$$

d) Convert $\frac{3\pi}{4}$ radians to degrees.

$$\frac{3\pi}{4} = \frac{3\pi}{4} \times \frac{180^\circ}{\pi} = 135^\circ$$

Arc Length

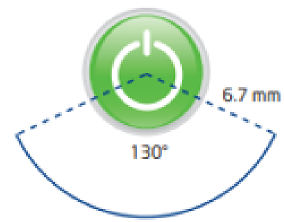
The radian measure of a central angle, θ, of a circle can also be defined as the ratio of the arc length to the radius of the circle.

Arc Length Equation: $\theta = \frac{a}{r}$ or $a = r\theta$

Note: To use this equation, the angle must be measured in radians.

Determine Arc Length in a Circle

Rosemarie is taking a course in industrial engineering. For an assignment, she is designing the interface of a DVD player. In her plan, she includes a decorative arc below the on/off button. The arc has central angle 130° in a circle with radius 6.7 mm. Determine the length of the arc, to the nearest tenth of a millimetre.



Convert the measure of the central angle to radians before using the formula $a = \theta r$, where a is the arc length; θ is the central angle, in radians; and r is the length of the radius.

$$180° = \pi$$
$$1° = \frac{\pi}{180}$$
$$130° = 130\left(\frac{\pi}{180}\right) = \frac{13\pi}{18}$$

$$a = \theta r = \left(\frac{13\pi}{18}\right)(6.7) = \frac{87.1\pi}{18} = 15.201\ldots$$

The arc length is 15.2 mm, to the nearest tenth of a millimetre.