Graphing Exponential Functions

What is an Exponential Function?

Exponential functions are one of the most important functions in mathematics. Exponential functions have many scientific applications, such as population growth and radioactive decay. Exponential function are also used in finance, so if you have a credit card, bank account, car loan, or home loan it is important to understand exponential functions and how they work.

Exponential functions are function where the variable x is in the exponent. Some examples of exponential functions are $f(x) = 2^x$, $f(x) = 5^{x-2}$, or $f(x) = 9^{2x+1}$. In each of the three examples the variable x is in the exponent, which makes each of the examples exponential functions.

An **Exponential Function** is a function of the form

$$f(x) = b^x \quad \text{or} \quad y = b^x$$

where b is called the "base" and b is a positive real number other than 1($b > 0$ and $b \neq 1$). The domain of an exponential function is all real numbers, that is, x can be any real number.

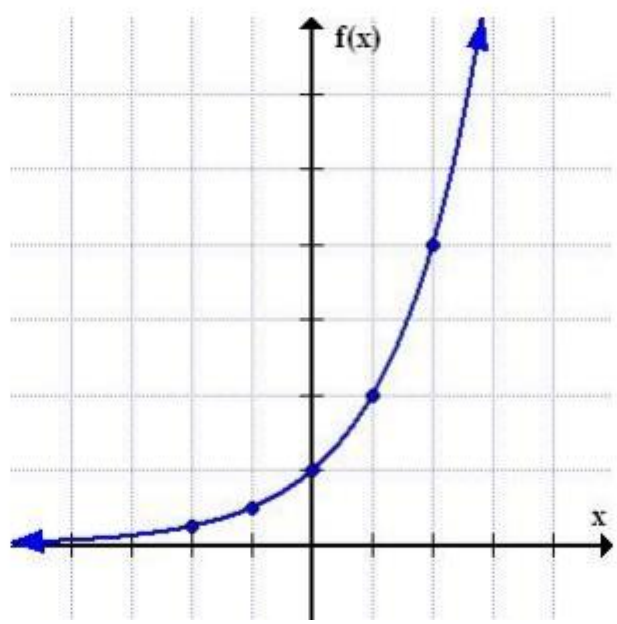
Graphing Exponential Functions

To begin graphing exponential functions we will start with two examples. We will graph the two exponential functions by making a table of values and plotting the points. After graphing the first two examples we will take a look at the similarities and differences between the two graphs.

When creating a table of values, I always suggest starting with the numbers x = –2, –1, 0, 1, and 2 because it is important to have different types of numbers, some negative, some positive, and zero.

Example 1: Graph $f(x) = 2^x$.

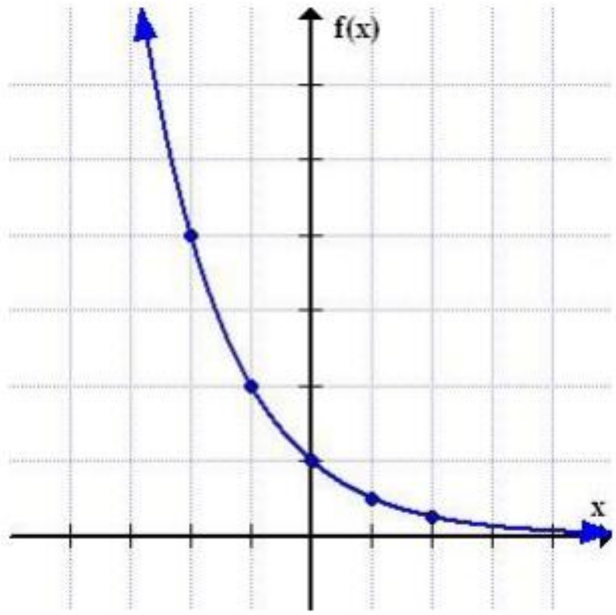
x	$f(x) = 2^x$
–2	$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
–1	$f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$f(0) = 2^0 = 1$
1	$f(1) = 2^1 = 2$
2	$f(2) = 2^2 = 4$



By plotting the five points in the table above and connecting the points, we get the graph shown above. Notice that as the x-values get smaller, x = –1, –2, etc. the graph of the function gets closer and closer to the x-axis, but never touches the x-axis. This means that there is a horizontal asymptote at the x-axis or y = 0. A horizontal asymptote is a horizontal line that the graph gets closer and closer to.

Example 2: Graph $f(x) = \left(\frac{1}{2}\right)^x$

x	$f(x) = \left(\frac{1}{2}\right)^x$
–2	$f(-2) = \left(\frac{1}{2}\right)^{-2} = \frac{1^{-2}}{2^{-2}} = \frac{2^2}{1^2} = 4$
–1	$f(-1) = \left(\frac{1}{2}\right)^{-1} = \frac{1^{-1}}{2^{-1}} = \frac{2^1}{1^1} = 2$
0	$f(0) = \left(\frac{1}{2}\right)^0 = 1$
1	$f(1) = \left(\frac{1}{2}\right)^1 = \frac{1^1}{2^1} = \frac{1}{2}$
2	$f(2) = \left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$



By plotting the five points in the table above and connecting the points, we get the graph shown above. Notice that as the x-values get larger, x = 1, 2, etc. the graph of the function gets closer and closer to the x-axis, but never touches the x-axis. This means that there is a horizontal asymptote at the x-axis or y = 0.

Now we can look at the similarities and differences between the graphs.

Similarities
- The domain for each example is all real numbers.
- The range for each example is all positive real numbers.
- Both graphs pass through the point (0, 1) or the y-intercept in each graph is 1.
- Both graphs get closer and closer to the x-axis, but do not touch the x-axis. So, each graph has a horizontal asymptote at the x-axis or y = 0.

Differences
- In Example 1, the graph goes upwards as it goes from left to right making it an increasing function. An exponential function that goes up from left to right is called "Exponential Growth".
- In Example 2, the graph goes downwards as it goes from left to right making it a decreasing function. An exponential function that goes down from left to right is called "Exponential Decay".