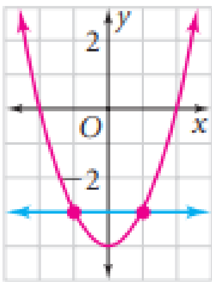
Solving Systems of Linear and Quadratic Equations

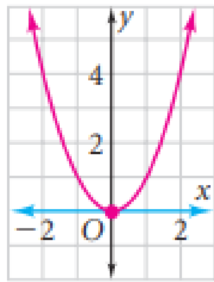
In this lesson, you will study systems of linear and quadratic equations. This type of system can have one solution, two solutions, or no solutions.

$y = x^2 - 4$
$y = -3$



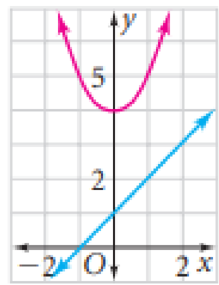
two solutions

$y = x^2$
$y = 0$



one solution

$y = x^2 + 4$
$y = x + 1$



no solutions

Solving System Using Algebraic Methods

EXAMPLE **Using Elimination**

Solve the following system of equations: $y = x^2 - 11x - 36$
$y = -12x + 36$

Step 1 Eliminate y.

$$\begin{array}{rl} y = x^2 - 11x - 36 & \\ -\ (y = \quad -12x + 36) & \textbf{Subtract the two equations.} \\ \hline 0 = x^2 + x - 72 & \textbf{Subtraction Property of Equality} \end{array}$$

Step 2 Factor and solve for x

$$\begin{array}{lll} 0 = x^2 + x - 72 & & \\ 0 = (x + 9)(x - 8) & & \textbf{Factor.} \\ x + 9 = 0 \quad \text{or} \quad x - 8 = 0 & & \textbf{Zero-Product Property} \\ x = -9 \quad \text{or} \quad x = 8 & & \end{array}$$

Step 3 Find the corresponding y values. Use either equation.

$$\begin{array}{ll} y = x^2 - 11x - 36 & y = x^2 - 11x - 36 \\ y = (-9)^2 - 11(-9) - 36 & y = (8)^2 - 11(8) - 36 \\ y = 81 + 99 - 36 & y = 64 - 88 - 36 \\ y = 144 & y = -60 \end{array}$$

The solutions are $(-9, 144)$ and $(8, -60)$.

EXAMPLE Using Substitution

Solve the following system of equations: $y = x^2 - 6x + 9$ and $y + x = 5$.

Step 1 Solve $y + x = 5$ for y.

$$y + x - x = 5 - x \qquad \textbf{Subtract } x \textbf{ from both sides.}$$
$$y = 5 - x$$

Step 2 Write a single equation containing only one variable.

$$y = x^2 - 6x + 9$$
$$5 - x = x^2 - 6x + 9 \qquad \textbf{Substitute } 5 - x \textbf{ for } y.$$
$$5 - x - (5 - x) = x^2 - 6x + 9 - (5 - x) \qquad \textbf{Subtract } 5 - x \textbf{ from both sides.}$$
$$0 = x^2 - 5x + 4$$

Step 3 Factor and solve for x.

$$0 = (x - 4)(x - 1) \qquad \textbf{Factor.}$$
$$x - 4 = 0 \quad \text{or} \quad x - 1 = 0 \qquad \textbf{Zero-Product Property}$$
$$x = 4 \quad \text{or} \quad x = 1$$

Step 4 Find the corresponding y-values. Use either equation.

$$y = -x^2 + 4x + 1 \qquad\qquad y = -x^2 + 4x + 1$$
$$= -(4^2) + 4(4) + 1 \qquad\qquad = -(1^2) + 4(1) + 1$$
$$= 1 \qquad\qquad = 4$$

The solutions of the system are $(4, 1)$ and $(1, 4)$.

EXAMPLE Using the Discriminant to Count Solutions

At how many points do the graphs of $y = 2$ and $y = x^2 + 4x + 7$ intersect?

Step 1 Eliminate y from the system. Write the resulting equation in standard form.

$$y = x^2 + 4x + 7$$
$$-\underline{(y = \qquad\qquad 2)} \qquad \textbf{Subtract the two equations.}$$
$$0 = x^2 + 4x + 5 \qquad \textbf{Subtraction Property of Equality}$$

Step 2 Determine whether the discriminant, $b^2 - 4ac$, is positive, 0, or negative.

$$b^2 - 4ac = 4^2 - 4(1)(5) \qquad \textbf{Evaluate the discriminant.}$$
$$= 16 - 20 \qquad \textbf{Use } a = 1, b = 4, \textbf{ and } c = 5.$$
$$= -4$$

Since the discriminant is -4, there are no solutions. The graphs do not intersect.