

Related Rates

In differential calculus, related rates problems involve finding a rate at which a quantity changes by relating that quantity to other quantities whose rates of change are known. The rate of change is usually with respect to time. Because science and engineering often relate quantities to each other, the methods of related rates have broad applications in these fields. Because problems involve several variables, differentiation with respect to time or one of the other variables requires application of the chain rule.

Procedure

The most common way to approach related rates problems is the following:

1. Identify the known variables, including rates of change and the rate of change that is to be found. (Drawing a picture or representation of the problem can help to keep everything in order.)
2. Construct an equation relating the quantities whose rates of change are known to the quantity whose rate of change is to be found.
3. Differentiate both sides of the equation with respect to time (or other rate of change).
Often, the chain rule is employed at this step.
4. Substitute the known rates of change and the known quantities into the equation.
5. Solve for the wanted rate of change.

Errors in this procedure are often caused by plugging in the known values for the variables before (rather than after) finding the derivative with respect to time. Doing so will yield an incorrect result, since if those values are substituted for the variables before differentiation, those variables will become constants; and when the equation is differentiated, zeroes appear in places of all variables for which the values were plugged in.