Pre-Calculus A 120

Notes and Sample Problems – Applications of Exponential Functions

Exponential Growth/Decay Formula: $y = ab^{\frac{t}{P}}$

Defining the variables

y represents the future amount (at time *t*)

a represents the initial amount

b represents the growth or decay factor

Note 1: If $b > 1$, the equation models exponential growth
If $0 < b < 1$, the equation models exponential decay

Note 2: When given a percentage of growth or decay, determine the growth/decay factor by adding or subtracting the percent, as a decimal, from 1.

Examples: an amount increases by 3% $b = 1 + 0.03 = 1.03$
an amount decreases by 40% $b = 1 - 0.40 = 0.60$

t represents the elapsed time
P represents the period for the growth/decay to occur

Problems

1. A bacterial culture doubles every 2 hours. If the culture started with 24 000 bacteria, how many bacteria will be present in 5 hours?

$y = a b^{\frac{t}{P}}$

$y = 24000(2)^{\frac{5}{2}}$

$y = 135765$

There will be 135765 bacteria in 5 hours.

2. The half-life of a radioactive sample is 4 hours. If 60 g of the sample was initially present, how much will remain after 7 hours? *Round your answer to the nearest hundredth of a gram.*

$$y = ab^{\frac{t}{p}}$$

$$y = 60\left(\frac{1}{2}\right)^{\frac{7}{4}}$$

$$y = 17.84$$

The amount of radioactive sample will be 17.84 g in 7 hours.

3. A bacteria culture triples every P hours. If the culture started with 13 000 bacteria, and there are 24 000 after 2 hours, what is the value of the period in hours? *Round your answer to the nearest hundredth of an hour.*

$$y = ab^{\frac{t}{p}}$$

$$24000 = 13000\,(3)^{\frac{2}{p}}$$

$$1.84615 = 3^{\frac{2}{p}}$$

Method 1

$$\log 1.84615 = \log 3^{\frac{2}{p}}$$

$$\log 1.84615 = \frac{2}{p}\log 3$$

$$p = \frac{2\log 3}{\log 1.84615}$$

$$p = 3.58$$

Method 2

$1.84615 = 3^{\frac{2}{p}}$

$\log_3 1.84615 = \frac{2}{p}$

$\frac{\log 1.84615}{\log 3} = \frac{2}{p}$

Use the change of base formula

$p = \frac{2 \log 3}{\log 1.84615}$

$p = 3.58$

The period is 3.58 hours.

4. The half-life of a radioactive sample is 6.2 hours. If 2000 g of the sample is present after 7 hours, how many grams of the sample was initially present? *Round your answer to the nearest gram.*

$y = ab^{\frac{t}{p}}$

$2000 = a\left(\frac{1}{2}\right)^{\frac{7}{6.2}}$

$2000 = a(0.457222)$

$a = \frac{2000}{0.457222}$

$a = 4374$

Initially, there was 4374 g of the radioactive sample.

6. The population of a town changes by an exponential growth factor, b, every 4 years. If the number of people grows from 2350 to 7000 in 3 years, what is the value of *b*? *Round your answer to 2 decimal places.*

$$y = ab^{\frac{t}{p}}$$

$$7000 = 2350\, b^{\frac{3}{4}}$$

$$\frac{7000}{2350} = b^{\frac{3}{4}}$$

$$2.9787 = b^{\frac{3}{4}}$$

$$(2.9787)^{\frac{4}{3}} = \left(b^{\frac{3}{4}}\right)^{\frac{4}{3}}$$

$$4.29 = b$$

The value of the growth factor is 4.29.