Adding and Subtracting Radicals

Adding and subtracting radicals is very similar to adding and subtracting with variables.

Examples

$$5x + 3x - 2x = 6x$$

$$5\sqrt{11} + 3\sqrt{11} - 2\sqrt{11} = 6\sqrt{11}$$

Only like radicals can be added or subtracted. If the indices and radicands are the same, then the terms in front of each like radical can be added or subtracted.

Example

$$\overbrace{7\sqrt[5]{6} + \underbrace{4\sqrt[5]{3} - \sqrt[5]{3}}_{\textit{Like Radicals}} + \sqrt[5]{6}}^{\textit{Like Radicals}} = 8\sqrt[5]{6} - 5\sqrt[5]{3}$$

Radicals may have to be simplified before they can be added or subtracted.

Example 1

$5\sqrt{45} + 6\sqrt{18} - 2\sqrt{98} + \sqrt{20}$	Simplify radicals
$= 5\sqrt{9 \cdot 5} + 6\sqrt{9 \cdot 2} - 2\sqrt{49 \cdot 2} + \sqrt{4 \cdot 5}$	
$= 5 \cdot 3\sqrt{5} + 6 \cdot 3\sqrt{2} - 2 \cdot 7\sqrt{2} + 2\sqrt{5}$	
$= 15\sqrt{5} + 18\sqrt{2} - 14\sqrt{2} + 2\sqrt{5}$	Combine like radicals
$= 17\sqrt{5} + 4\sqrt{2}$	Final answer

Example 2

$$
\begin{aligned}
& 4\sqrt[3]{54} - 9\sqrt[3]{16} + 5\sqrt[3]{9} && \text{Simplify radicals} \\
&= 4\sqrt[3]{27\cdot 2} - 9\sqrt[3]{8\cdot 2} + 5\sqrt[3]{9} \\
&= 4\cdot 3\sqrt[3]{2} - 9\cdot 2\sqrt[3]{2} + 5\sqrt[3]{9} \\
&= \underbrace{12\sqrt[3]{2} - 18\sqrt[3]{2}}_{\textit{Like Radicals}} + 5\sqrt[3]{9} && \text{Combine like terms} \\
&= -6\sqrt[3]{2} + 5\sqrt[3]{9} && \text{Final answer}
\end{aligned}
$$

Multiplying Radicals

Radicals may be multiplied together as long as their indices are the same. Multiply the factors outside the radical together and mulitply the radicands.

Product Rule of Radicals

$$a\sqrt[n]{b} \cdot c\sqrt[n]{d} = ac\sqrt[n]{bd}$$

Example 1

$$
\begin{aligned}
& -5\sqrt{14}\cdot 4\sqrt{6} && \text{Mulitply outside and inside the radical} \\
&= -20\sqrt{84} && \text{Simplify the radical} \\
&= -20\sqrt{4\cdot 21} \\
&= -20\cdot 2\sqrt{21} \\
&= -40\sqrt{21}
\end{aligned}
$$

Example 2

$$
\begin{aligned}
& 2\sqrt[3]{18}\cdot 6\sqrt[3]{15} && \text{Multiply outside and inside the radical} \\
&= 12\sqrt[3]{270} && \text{Simplify the radical} \\
&= 12\sqrt[3]{27\cdot 10} \\
&= 12\cdot 3\sqrt[3]{10} \\
&= 36\sqrt[3]{10}
\end{aligned}
$$

When multiplying with radicals we can still use the distributive property or FOIL just as we would with variables.

Example 1

$7\sqrt{6}(3\sqrt{10}-5\sqrt{15})$	Distribute
$= 21\sqrt{60}-35\sqrt{90}$	Simplify each radical
$= 21\sqrt{4\cdot 15}-35\sqrt{9\cdot 10}$	
$= 21\cdot 2\sqrt{15}-35\cdot 3\sqrt{10}$	
$= 42\sqrt{15}-105\sqrt{10}$	

Example 2

$(\sqrt{5}-2\sqrt{3})(4\sqrt{10}+6\sqrt{6})$	Foil
$= 4\sqrt{50}+6\sqrt{30}-8\sqrt{30}-12\sqrt{18}$	Simplify
$= 4\sqrt{25\cdot 2}-2\sqrt{30}-12\sqrt{9\cdot 2}$	
$= 4\cdot 5\sqrt{2}-2\sqrt{30}-12\cdot 3\sqrt{2}$	
$= 20\sqrt{2}-2\sqrt{30}-36\sqrt{2}$	Add like radicals
$= -16\sqrt{2}-2\sqrt{30}$	

Example 3

$(2\sqrt{5}-3\sqrt{6})(7\sqrt{2}-8\sqrt{7})$	Foil
$= 14\sqrt{10}-16\sqrt{35}-21\sqrt{12}+24\sqrt{42}$	Simplify radicals
$= 14\sqrt{10}-16\sqrt{35}-21\sqrt{4\cdot 3}+24\sqrt{42}$	
$= 14\sqrt{10}-16\sqrt{35}-21\cdot 2\sqrt{3}+24\sqrt{42}$	
$= 14\sqrt{10}-16\sqrt{35}-42\sqrt{3}+24\sqrt{42}$	