Adding and Subtracting Radicals

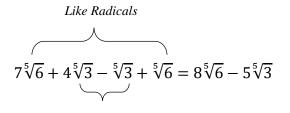
Adding and subtracting radicals is very similar to adding and subtracting with variables.

Examples

5x + 3x - 2x = 6x $5\sqrt{11} + 3\sqrt{11} - 2\sqrt{11} = 6\sqrt{11}$

Only like radicals can be added or subtracted. If the indices and radicands are the same, then the terms in front of each like radical can be added or subtracted.

Example





Radicals may have to be simplified before they can be added or subtracted.

Example 1

$$5\sqrt{45} + 6\sqrt{18} - 2\sqrt{98} + \sqrt{20}$$

Simplify radicals
$$= 5\sqrt{9 \cdot 5} + 6\sqrt{9 \cdot 2} - 2\sqrt{49 \cdot 2} + \sqrt{4 \cdot 5}$$

$$= 5 \cdot 3\sqrt{5} + 6 \cdot 3\sqrt{2} - 2 \cdot 7\sqrt{2} + 2\sqrt{5}$$

$$= 15\sqrt{5} + 18\sqrt{2} - 14\sqrt{2} + 2\sqrt{5}$$

Combine like radicals
$$= 17\sqrt{5} + 4\sqrt{2}$$

Final answer

Example 2

$$4\sqrt[3]{54} - 9\sqrt[3]{16} + 5\sqrt[3]{9}$$

Simplify radicals
$$= 4\sqrt[3]{27 \cdot 2} - 9\sqrt[3]{8 \cdot 2} + 5\sqrt[3]{9}$$

$$= 4 \cdot 3\sqrt[3]{2} - 9 \cdot 2\sqrt[3]{2} + 5\sqrt[3]{9}$$

$$= 12\sqrt[3]{2} - 18\sqrt[3]{2} + 5\sqrt[3]{9}$$

Combine like terms
Like Radicals

$$= -6\sqrt[3]{2} + 5\sqrt[3]{9}$$
 Final answer

Multiplying Radicals

Radicals may be multiplied together as long as their indices are the same. Multiply the factors outside the radical together and multiply the radicands.

Product Rule of Radicals

$$a\sqrt[n]{b} \cdot c\sqrt[n]{d} = ac\sqrt[n]{bd}$$

Example 1

$$-5\sqrt{14} \cdot 4\sqrt{6}$$
$$= -20\sqrt{84}$$
$$= -20\sqrt{4 \cdot 21}$$
$$= -20 \cdot 2\sqrt{21}$$
$$= -40\sqrt{21}$$

Mulitply outside and inside the radical Simplify the radical

Example 2

$$2\sqrt[3]{18} \cdot 6\sqrt[3]{15}$$

= $12\sqrt[3]{270}$
= $12\sqrt[3]{27 \cdot 10}$
= $12 \cdot 3\sqrt[3]{10}$
= $36\sqrt[3]{10}$

Multiply outside and inside the radical Simplify the radical

When multiplying with radicals we can still use the distributive property or FOIL just as we would with variables.

Example 1

$$7\sqrt{6}(3\sqrt{10} - 5\sqrt{15})$$

= $21\sqrt{60} - 35\sqrt{90}$
= $21\sqrt{4 \cdot 15} - 35\sqrt{9 \cdot 10}$
= $21 \cdot 2\sqrt{15} - 35 \cdot 3\sqrt{10}$
= $42\sqrt{15} - 105\sqrt{10}$

Distribute Simplify each radical

Example 2

$(\sqrt{5} - 2\sqrt{3})(4\sqrt{10} + 6\sqrt{6})$	Foil
$= 4\sqrt{50} + 6\sqrt{30} - 8\sqrt{30} - 12\sqrt{18}$	Simplify
$= 4\sqrt{25 \cdot 2} - 2\sqrt{30} - 12\sqrt{9 \cdot 2}$	
$= 4 \cdot 5\sqrt{2} - 2\sqrt{30} - 12 \cdot 3\sqrt{2}$	
$= 20\sqrt{2} - 2\sqrt{30} - 36\sqrt{2}$	Add like radicals
$= -16\sqrt{2} - 2\sqrt{30}$	

Example 3

$(2\sqrt{5} - 3\sqrt{6})(7\sqrt{2} - 8\sqrt{7})$	Foil
$= 14\sqrt{10} - 16\sqrt{35} - 21\sqrt{12} + 24\sqrt{42}$	Simplify radicals
$= 14\sqrt{10} - 16\sqrt{35} - 21\sqrt{4\cdot 3} + 24\sqrt{42}$	
$= 14\sqrt{10} - 16\sqrt{35} - 21 \cdot 2\sqrt{3} + 24\sqrt{42}$	
$= 14\sqrt{10} - 16\sqrt{35} - 42\sqrt{3} + 24\sqrt{42}$	