# Position, Velocity and Acceleration Notes

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We can describe the position of an object by an equation, $s = f(t)$; where ***s* is the position of the object at time *t*.**

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The **velocity** of a moving particle is represented by the derivative of *s*: Velocity $= v(t) = s'(t)$.
(The units for velocity are typically: meters per second (m/s) or feet per second (ft/sec))

Velocity tells us both the speed of the object and the direction in which the object is travelling.

- If velocity $= 0$, then the object is not moving
- If velocity $> 0$, then the object is moving forward, to the right or up
- If velocity $< 0$, then the object is moving backward, to the left or down
- Speed is absolute value of the velocity: speed $= |v(t)|$

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The acceleration of a moving object is represented by the derivative of *v*:
Acceleration $= a(t) = v'(t) = s''(t)$

- If acceleration $= 0$, then the velocity is not changing
- If acceleration $> 0$, then velocity is increasing
- If acceleration $< 0$, then velocity is decreasing

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An object is **speeding up** when the **velocity** and **acceleration** are the **SAME** signs!
An object is **slowing down** when the **velocity** and **acceleration** are **OPPOSITE** signs!

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Displacement over time = Displacement $= s(\text{end time}) - s(\text{beginning time}) = s(b) - s(a)$

Average velocity = $\dfrac{1}{b-a}\int_a^b v(t)\,dt = \dfrac{s(b)-s(a)}{b-a} = \dfrac{\text{displacement}}{\text{time}}$

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**Ex. 2 If a ball is thrown vertically upward from the top of an 80 ft. tower with an initial velocity of 72 ft/sec, then its height, *h*, above the ground *t* seconds later is given by the equation** $h(t) = 80 + 72t - 16t^2$

a) **Find the greatest height reached by the ball.**
b) **How fast is the ball traveling when it reaches a height of 136 ft?**

a) The greatest height occurs when the ball "stops" and begins to fall back to Earth. When the ball stops, what is its velocity? It is zero. Therefore, we need to find the velocity equation and set it equal to zero so we know **when** it reaches maximum height.

$$v = 72 - 32t$$

$$72 - 32t = 0$$

Lets find $t$: $72 = 32t$

$$t = \frac{72}{32} = \frac{9}{4}$$

We know when it reaches its maximum height, so substitute that in for $t$ and find $h$.

$h\left(\frac{9}{4}\right) = 80 + 72\left(\frac{9}{4}\right) - 16\left(\frac{9}{4}\right)^2 = 161$ ft. The ball reaches a max height of 161 ft.

b) The ball is going to reach the height of 136 ft. two times: once on the way up and once on the way down. Again, we need to know **when** this occurs. We need to substitute 136 in for $h$ and then solve for $t$.

$$136 = 80 + 72t - 16t^2$$

$$16t^2 - 72t + 56 = 0$$

$$2t^2 - 9t + 7 = 0$$

$$(2t - 7)(t - 1) = 0$$

$$t = \frac{7}{2}; \quad t = 1$$

This means that at $t = 1$ seconds the ball is 136 ft. above the ground (on the way up) and at $t = 7/2$ seconds the ball is 136 ft. above the ground (on the way down).

To calculate how fast the ball is traveling when it is 136 ft. above the ground, calculate the velocity at $t = 1$ and $t = 7/2$.

$$v(t) = 72 - 32t$$

$$v(1) = 72 - 32(1) = 40 \text{ ft/sec}$$

$$v\left(\frac{7}{2}\right) = 72 - 32\left(\frac{7}{2}\right) = -40 \text{ ft/sec}$$

Notice at $t = 1$, velocity is positive… That means the ball is traveling up at 40 ft/sec.
Notice at $t = 7/2$, velocity is negative… That means the ball is traveling down at 40 ft.sec