Radical Equations

In order to solve a radical equation we must raise both sides of the equation to an exponent. For example, to clear a square root, square both sides of the equation.

Sometimes when solving a radical equation, it is possible to get a value for the variable that is not an answer to the original equation. A value that does not work is called an extraneous solution. Therefore, all possible solutions must be checked in the original equation.

Example 1

$\sqrt{7x+2} = 4$ Square both sides of the equation.

$\left(\sqrt{7x+2}\right)^2 = 4^2$

$7x + 2 = 16$ Solve the linear equation.

$7x = 16 - 2$

$7x = 14$

$x = 2$ Possible solution.

Restriction on Variable

$$7x + 2 \geq 0$$

$$7x \geq -2$$

$$x \geq \frac{-2}{7}$$

Check $x = 2$ in the original equation.

Left Side	Right Side
$\sqrt{7x+2}$ $= \sqrt{7(2)+2}$ $= \sqrt{14+2}$ $= \sqrt{16}$ $= 4$	4

LS = RS

The solution is $x = 2$.

Example 2

$$\sqrt{3x-2}+11=0 \qquad \text{Isolate the radical.}$$
$$\sqrt{3x-2}=\underbrace{\mathbf{-11}}_{\textit{Negative Number!}}$$

Restriction on Variable
$3x-2\geq 0$
$3x\geq 2$
$x\geq \frac{2}{3}$

This radical equation has no solution because a square root cannot equal a negative number.

Example 3

$x+\sqrt{4x+1}=5$	Isolate the radical.
$\sqrt{4x+1}=5-x$	Square both sides of the equation.
$\left(\sqrt{4x+1}\right)^2=(5-x)^2$	
$4x+1=25-10x+x^2$	Solve the quadratic equation.
$0=x^2-10x-4x+25-1$	
$0=x^2-14x+24$	
$0=(x-12)(x-2)$	
$x=12 \qquad x=2$	Possible solutions.

Restriction on Variable
$4x+1\geq 0$
$4x\geq -1$
$x\geq \frac{-1}{4}$

Check $x=12$ in the original equation.

Left Side	Right Side
$x+\sqrt{4x+1}$ $=12+\sqrt{4(12)+1}$ $=12+\sqrt{48+1}$ $=12+\sqrt{49}$ $=12+7$ $=19$	5

LS $\neq$ RS

$x=12$ *is an extraneous solution*

Check $x = 2$ in the original equation.

Left Side	**Right Side**
$x + \sqrt{4x + 1}$ $= 2 + \sqrt{4(2) + 1}$ $= 2 + \sqrt{8 + 1}$ $= 2 + \sqrt{9}$ $= 2 + 3$ $= 5$	5

LS = RS

The solution is $x = 2$.