Rationalizing Denominators

It is considered bad practice to have a radical in the denominator of a fraction. The process of clearing the denominator of a radical is called rationalizing the denominator.

Reminder: When a radical is multiplied by itsef, the product equals the radicand.

Examples $\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$

$$\sqrt{7} \cdot \sqrt{7} = \sqrt{49} = 7$$

Monomial Denominators

To clear the denominator of a radical, multiply both the numerator and denominator by the radical in the denominator.

Example 1

$$\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$ Multiply the numerator and denominator by $\sqrt{2}$.

$$= \frac{3\sqrt{2}}{\sqrt{4}}$$

$$= \frac{3\sqrt{2}}{2}$$

Example 2

$$\frac{-2}{5\sqrt{3}}$$ Mulitply the numerator and denominator by $\sqrt{3}$.

$$= \frac{-2}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{-2\sqrt{3}}{5\sqrt{9}}$$

$$= \frac{-2\sqrt{3}}{5 \cdot 3}$$

$$= \frac{-2\sqrt{3}}{15}$$

Example 3

$$\frac{\sqrt{3}-9}{2\sqrt{6}}$$ Multiply the numerator and denominator by $\sqrt{6}$.

$$= \frac{\sqrt{3}-9}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$ Multiply the numerators and denominators.

$$= \frac{\sqrt{18}-9\sqrt{6}}{2\sqrt{36}}$$ Simplify $\sqrt{18}$ and $\sqrt{36}$.

$$= \frac{\sqrt{9\cdot 2}-9\sqrt{6}}{2\cdot 6}$$

$$= \frac{3\sqrt{2}-9\sqrt{6}}{12}$$ Cancel the common factor, 3, from each term.

$$= \frac{\sqrt{2}-3\sqrt{6}}{4}$$

As you are rationalizing, it is always important to check your expression to see if it can be simplified more. Ask yourself, can the fraction be reduced? can the radicals be simplified?

Binomial Denominators

If a binomial occurs in the denominator, a different strategy must be used to clear the the denominator or radicals.

Reminders:

1. Conjugates are binomials that have the same terms, but opposite signs in the middle.

   Examples $x - 3$ and $x + 3$ are conjugates

   $2x + 5$ and $2x - 5$ are conjugates

   $6 - \sqrt{3}$ and $6 + \sqrt{3}$ are conjugates

   $4\sqrt{7} + 1$ and $4\sqrt{7} - 1$ are conjugates

2. When conjugates are multiplied together, the product is a difference of squares.

Examples $(x-2)(x+2) = x^2 - 4$

$(3x+4)(3x-4) = 9x^2 - 16$

$(\sqrt{5}+2)(\sqrt{5}-2) = \sqrt{25} - 4 = 5 - 4 = 1$

$(4\sqrt{3}-6)(4\sqrt{3}+6) = 16\sqrt{9} - 36 = 16 \cdot 3 - 36 = 48 - 36 = 12$

3. When binomial conjugates containing radicals are multiplied together, the product contains no radicals – *see above*.

Example 1

$\frac{2}{\sqrt{3}-5}$ Multiply the numerator and denominator by the conjugate of $\sqrt{3}-5$.

$= \frac{2}{\sqrt{3}-5} \cdot \frac{\sqrt{3}+5}{\sqrt{3}+5}$ Multiply the numerators and denominators.

$= \frac{2\sqrt{3}+10}{\sqrt{9}-25}$ Simplify the denominator.

$= \frac{2\sqrt{3}+10}{3-25}$

$= \frac{2\sqrt{3}+10}{-22}$ Cancel the common factor -2 from each term.

$= \frac{-\sqrt{3}-10}{11}$

Example 2

$\frac{3-\sqrt{5}}{2-\sqrt{3}}$

Multiply the numerator and denominator by the conjugate of $2-\sqrt{3}$.

$= \frac{3-\sqrt{5}}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$

Multiply the numerators and the denominators.

$= \frac{6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}}{4-\sqrt{9}}$

Simplify the denominator.

$= \frac{6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}}{4-3}$

$= 6+3\sqrt{3}-2\sqrt{5}-\sqrt{15}$