

Derivatives of Trigonometric Functions

The complete list of derivatives of trigonometric functions:

$$1. \frac{d}{dx} \sin x = \cos x$$

$$2. \frac{d}{dx} \cos x = -\sin x$$

$$3. \frac{d}{dx} \tan x = \sec^2 x$$

$$4. \frac{d}{dx} \sec x = \sec x \tan x$$

$$5. \frac{d}{dx} \cot x = -\csc^2 x$$

$$6. \frac{d}{dx} \csc x = -\csc x \cot x$$

Examples

ex. Differentiate $f(x) = \sec x + 5 \csc x$

$$f'(x) = \sec x \tan x + 5(-\csc x \cot x) = \sec x \tan x - 5 \csc x \cot x$$

ex. Differentiate $f(x) = x^2 \cos x - 2x \sin x - 3 \cos x$

$$f'(x) = [x^2(-\sin x) + (2x) \cos x] - 2[x(\cos x) + (1)\sin x] - 3(-\sin x)$$

$$= -x^2 \sin x + 2x \cos x - 2x \cos x - 2\sin x + 3\sin x$$

$$= -x^2 \sin x + \sin x$$

ex. Differentiate $s(t) = \frac{\sin t}{1 - \cos t}$

$$\begin{aligned} s'(t) &= \frac{(1 - \cos t)(\cos t) - (\sin t)(0 - (-\sin t))}{(1 - \cos t)^2} \\ &= \frac{\cos t - \cos^2 t - \sin^2 t}{(1 - \cos t)^2} = \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1 - \cos t)^2} \\ &= \frac{\cos t - 1}{(1 - \cos t)^2} = \frac{-(1 - \cos t)}{(1 - \cos t)^2} = \frac{-1}{1 - \cos t} = \frac{1}{\cos t - 1} \end{aligned}$$