Find two positive numbers whose sum is 36 and whose product is a maximum.

Primary

$$P = xy$$

$$P(x) = x(36 - x)$$

$$P'(x) = 36 - 2x$$

$$36 - 2x = 0$$

$$x = 18$$

Secondary

$$x + y = 36$$

$$y = 36 - x$$

$$y = 36 - 18 = 18$$

$$(18, 18)$$

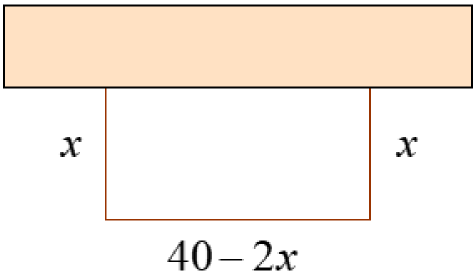
Intervals:	$(0, 18)$	$(18, 36)$
Test values:	1	20
P'(test pt)	$+$	$-$
$P(x)$	inc	dec

rel max

$x = 18$

A Classic Problem

You have 40 feet of fence to enclose a rectangular garden along the side of a barn. What is the maximum area that you can enclose?



$$A = x(40 - 2x)$$

$$A = 40x - 2x^2$$

$$A' = 40 - 4x$$

$$0 = 40 - 4x$$

$$4x = 40$$

$$x = 10$$

$$A = 10(40 - 2 \cdot 10)$$

$$A = 10(20)$$

$$\boxed{A = 200 \text{ ft}^2}$$

$$w = x$$

$$l = 40 - 2x$$

$$\boxed{w = 10 \text{ ft}, \quad l = 20 \text{ ft}}$$

An open box having a square base and a surface area of 108 square inches is to have a maximum volume. Find its dimensions.

Primary

$$V = x^2 y$$

$$V(x) = x^2\left(\frac{108 - x^2}{4x}\right)$$

$$= \frac{108x}{4} - \frac{x^3}{4}$$

$$= 27x - \tfrac{1}{4}x^3$$

$$V'(x) = 27 - \tfrac{3}{4}x^2$$

$$27 - \tfrac{3}{4}x^2 = 0$$

$$x^2 = -27\left(-\tfrac{4}{3}\right)$$

$$x = \pm 6$$

Secondary

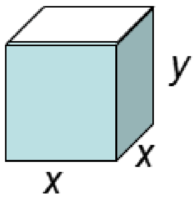
$$SA = x^2 + 4xy$$

$$108 = x^2 + 4xy$$

$$\frac{108 - x^2}{4x} = y$$

$$\frac{108 - (6)^2}{4(6)} = y$$

$$3 = y$$



Domain of x will range from x being as small as possible to x as large as possible.

Smallest	Largest
(x is near zero)	(y is near zero)
$x \approx 0$	$x^2 \approx 108$

Intervals:	$(0, 6)$	$(6, \sqrt{108})$
Test values:	1	10
V'(test pt)	$+$	$-$
V(x)	inc	dec

rel max

$x = 6$

Dimensions: 6 in x 6 in x 3 in

Find the point on $f(x) = x^2$ that is closest to (0,3).

Minimize distance

Primary

$$d = \sqrt{(x-0)^2 + (y-3)^2}$$

$$d = \sqrt{(x)^2 + (y-3)^2}$$

***The value of the root will be smallest when what is inside the root is smallest.

$$d = (x)^2 + (y-3)^2$$

$$d(x) = (x)^2 + (x^2-3)^2$$

$$d(x) = x^2 + x^4 - 6x^2 + 9$$

$$d(x) = x^4 - 5x^2 + 9$$

$$d'(x) = 4x^3 - 10x$$

$$4x^3 - 10x = 0$$

$$2x(2x^2 - 5) = 0$$

$$x = 0 \quad 2x^2 - 5 = 0$$

$$x = 0 \quad x = \pm\sqrt{\tfrac{5}{2}}$$

Secondary

$$y = x^2$$

Intervals:	$\left(-\infty, -\sqrt{\frac{5}{2}}\right)$	$\left(-\sqrt{\frac{5}{2}}, 0\right)$	$\left(0, \sqrt{\frac{5}{2}}\right)$	$\left(\sqrt{\frac{5}{2}}, \infty\right)$
Test values:	-3	-1	1	3
d '(test pt)	$-$	$+$	$-$	$+$
d(x)	dec	inc	dec	inc

rel min $x = -\sqrt{\frac{5}{2}}$ rel max   rel min $x = \sqrt{\frac{5}{2}}$

$$\left(-\sqrt{\frac{5}{2}}, \frac{5}{2}\right) \qquad \left(\sqrt{\frac{5}{2}}, \frac{5}{2}\right)$$

A rectangular page is to contain 24 square inches of print. The margins at the top and bottom are 1.5 inches. The margins on each side are 1 inch. What should the dimensions of the print be to use the least paper?

Primary

$$A=(x+2)(y+3)$$

$$A(x)=(x+2)\left(\tfrac{24}{x}+3\right)$$

$$=24+3x+\tfrac{48}{x}+6$$

$$=3x+48x^{-1}+30$$

$$A'(x)=3-\tfrac{48}{x^2}$$

$$=\frac{3x^2-48}{x^2}$$

crit #'s: $x=0,\pm4$

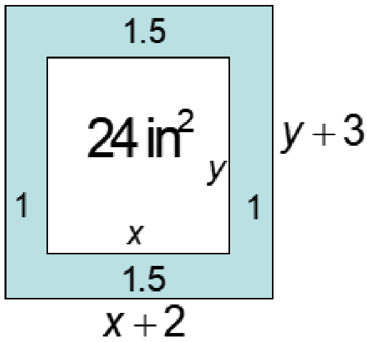
Secondary

$$xy=24$$

$$y=\tfrac{24}{x}$$

$$y=\tfrac{24}{(4)}$$

$$y=6$$



Smallest	Largest
(x is near zero)	(y is near zero)
$x\approx 0$	$x\approx 24$

Intervals:	$(0,4)$	$(4,24)$
Test values:	1	10
A '(test pt)	$-$	$+$
$A(x)$	dec	inc

rel min $x=4$

Print dimensions: 6 in x 4 in

Page dimensions: 9 in x 6 in