

## Solving Absolute Value Equations

Solving **absolute value equations** is as easy as working with regular linear equations. The only additional key step that you need to remember is to separate the original absolute value equation into two parts: positive and negative (+) components.

Below is the general approach on how to break them down into two equations:

$$|x| = a \begin{cases} x = +a \\ x = -a \end{cases}$$

### Examples

1. Solve the absolute value equation  $|x - 5| = 3$ .

$$|x - 5| = 3$$

$$\begin{array}{l|l} \boxed{x - 5 = 3} & \boxed{x - 5 = -3} \\ \hline x - 5 = 3 & x - 5 = -3 \\ x - 5 + 5 = 3 + 5 & x - 5 + 5 = -3 + 5 \\ x = 8 & x = 2 \end{array}$$

Check your answers using a LS/RS chart and the original absolute value equation.

2. Solve the absolute value equation  $|-2x + 7| = 25$ .

$$|-2x + 7| = 25$$

$$\begin{array}{l|l} \boxed{-2x + 7 = 25} & \boxed{-2x + 7 = -25} \\ \hline -2x + 7 = 25 & -2x + 7 = -25 \\ -2x + 7 - 7 = 25 - 7 & -2x + 7 - 7 = -25 - 7 \\ -2x = 18 & -2x = -32 \\ \frac{-2x}{-2} = \frac{18}{-2} & \frac{-2x}{-2} = \frac{-32}{-2} \\ x = -9 & x = 16 \end{array}$$

Check your answers using a LS/RS chart and the original absolute value equation.

3. Solve the equation  $3 + |x + 4| = 0$ .

$$\begin{array}{r} 3 + |x + 4| = 0 \\ -3 \quad \quad -3 \\ \hline |x + 4| = -3 \end{array}$$

This equation has no solution.

4. Solve the absolute value equation  $|-6x + 3| - 7 = 20$ .

$$|-6x + 3| - 7 = 20$$

$$|-6x + 3| - 7 + 7 = 20 + 7$$

$$|-6x + 3| = 27$$

$$\begin{array}{l|l} \boxed{-6x + 3 = 27} & \boxed{-6x + 3 = -27} \\ \hline -6x + 3 = 27 & -6x + 3 = -27 \\ -6x + 3 - 3 = 27 - 3 & -6x + 3 - 3 = -27 - 3 \\ -6x = 24 & -6x = -30 \\ \frac{-6x}{-6} = \frac{24}{-6} & \frac{-6x}{-6} = \frac{-30}{-6} \\ x = -4 & x = 5 \end{array}$$

Check your answers using a LS/RS chart and the original absolute value equation.

5. Solve the absolute value equation  $|x^2 + 2x - 4| = 4$ .

$$|x^2 + 2x - 4| = 4$$

$$\begin{array}{l|l} \boxed{x^2 + 2x - 4 = 4} & \boxed{x^2 + 2x - 4 = -4} \\ \hline x^2 + 2x - 4 = 4 & x^2 + 2x - 4 = -4 \\ x^2 + 2x - 4 - 4 = 4 - 4 & x^2 + 2x - 4 + 4 = -4 + 4 \\ x^2 + 2x - 8 = 0 & x^2 + 2x = 0 \\ (x + 4)(x - 2) = 0 & x(x + 2) = 0 \\ x = -4, 2 & x = -2, 0 \end{array}$$

Check your answers using a LS/RS chart and the original absolute value equation.

### Why do you need to check your answers?

Example of an absolute value equation with an extraneous root.

6. Solve the absolute value equations  $|3x + 2| = 4x + 5$ .

Case 1:	Case 2:	Answer: $x = -1$
$ 3x + 2  = 4x + 5$ $3x + 2 = 4x + 5$ $2 = x + 5$ $-3 = -3$	$ 3x + 2  = 4x + 5$ $3x + 2 = -(4x + 5)$ $3x + 2 = -4x - 5$ $7x = -7$ $x = -1$	The check shows that $x = -3$ is NOT a solution to this absolute value equation. There is only ONE answer: $x = -1$ . <b>Always check!!!!</b>
Check: $ 3(-1) + 2  = 4(-1) + 5$ $ 3(-1) + 2  = -4 + 5$ $ 3(-1) + 2  = 1$ $1 = 1$ ✓ No extraneous!	Check: $ 3(-1) + 2  = 4(-1) + 5$ $ 3(-1) + 2  = -4 + 5$ $ 3(-1) + 2  = 1$ $1 = 1$	