Solving Absolute Value Equations

Solving **absolute value equations** is as easy as working with regular linear equations. The only additional key step that you need to remember is to separate the original absolute value equation into two parts: positive and negative (±) components.

Below is the general approach on how to break them down into two equations:

$$|x| = a \quad \begin{cases} x = +a \\ x = -a \end{cases}$$

Examples

1. Solve the absolute value equation $|x - 5| = 3$.

$$|x-5|=3$$

$x-5=3$	$x-5=-3$
$x-5=3$	$x-5=-3$
$x-5+5=3+5$	$x-5+5=-3+5$
$x=8$ ✓	$x=2$ ✓

Check your answers using a LS/RS chart and the original absolute value equation.

2. Solve the absolute value equation $|-2x + 7| = 25$.

$$|-2x+7|=25$$

$-2x+7=25$	$-2x+7=-25$
$-2x+7=25$	$-2x+7=-25$
$-2x+7-7=25-7$	$-2x+7-7=-25-7$
$-2x=18$	$-2x=-32$
$\frac{-2x}{-2}=\frac{18}{-2}$	$\frac{-2x}{-2}=\frac{-32}{-2}$
$x=-9$ ✓	$x=16$ ✓

Check your answers using a LS/RS chart and the original absolute value equation.

3. Solve the equation $3 + |x + 4| = 0$.

$$\begin{array}{r} 3 + |x + 4| = 0 \\ \underline{-3 \qquad\quad -3} \\ |x + 4| = -3 \end{array}$$ ✗

This equation has no solution.

4. Solve the absolute value equation $|-6x + 3| - 7 = 20$.

$$|-6x+3|-7=20$$
$$|-6x+3|-7+7=20+7$$
$$|-6x+3|=27$$

$-6x+3=27$	$-6x+3=-27$
$-6x+3=27$	$-6x+3=-27$
$-6x+3-3=27-3$	$-6x+3-3=-27-3$
$-6x=24$	$-6x=-30$
$\frac{-6x}{-6}=\frac{24}{-6}$	$\frac{-6x}{-6}=\frac{-30}{-6}$
$x=-4$ ✓	$x=5$ ✓

Check your answers using a LS/RS chart and the original absolute value equation.

5. Solve the absolute value equation $|x^2 + 2x - 4| = 4$.

$$|x^2+2x-4|=4$$

$x^2+2x-4=4$	$x^2+2x-4=-4$
$x^2+2x-4=4$	$x^2+2x-4=-4$
$x^2+2x-4-4=4-4$	$x^2+2x-4+4=-4+4$
$x^2+2x-8=0$	$x^2+2x=0$
$(x+4)(x-2)=0$	$x(x+2)=0$
$x=-4,2$ ✓	$x=-2,0$ ✓

Check your answers using a LS/RS chart and the original absolute value equation.

Why do you need to check your answers?

Example of an absolute value equation with an extraneous root.

6. Solve the absolute value equations $|3x + 2| = 4x + 5$.