Simplifying Radicals

Expressing a radical in **simplest form** means:

- the radicand does not contain a factor that can be removed (there are no square roots, cube roots, 4th roots, etc. left under the root sign)
- there are no radicals in the denominator of a fraction.

Square Roots

Square roots are the most common type of radical. A square root "un-squares" a number.

Examples

$\sqrt{1} = 1$	$\sqrt{121} = 11$
$\sqrt{4} = 2$	$\sqrt{625} = 25$
$\sqrt{9} = 3$	$\sqrt{-81}$ is undefined

Note – Any number times itself is always a positive number (or zero), so you can't ever get a negative number by squaring. Since square roots undo squaring, negative numbers can't have square roots.

Not all numbers have a nice even square root. For example, $\sqrt{8} = 2.8284271247$ ...
To be as accurate as possible, we will express square roots in simplest radical form. To do this, we can use the Product Rule of Square Roots.

Product Rule of Square Roots

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

The fastest way to simplify a square root, is to find the greatest perfect square number that divides evenly into the radicand.

Example 1

$\sqrt{72}$	72 is divisible by 36, a perfect square
$= \sqrt{36 \cdot 2}$	Split into factors
$= \sqrt{36} \cdot \sqrt{2}$	Product Rule, take the square root of 36
$= 6\sqrt{2}$	Simplified radical

Example 2

$5\sqrt{63}$	63 is divisible by 9, a perfect square
$= 5\sqrt{9 \cdot 7}$	Split into factors
$= 5\sqrt{9} \cdot \sqrt{7}$	Product Rule, take the square root of 9
$= 5 \cdot 3\sqrt{7}$	Multiply coefficients
$= 15\sqrt{7}$	Simplified radical

Example 3

$\frac{9}{4}\sqrt{48}$	48 is divisible by 16, a perfect square
$= \frac{9}{4}\sqrt{16 \cdot 3}$	Split into factors
$= \frac{9}{4}\sqrt{16} \cdot \sqrt{3}$	Product Rule, take the square root of 16
$= \frac{9}{4} \cdot 4\sqrt{3}$	Multiply coefficients
$= 9\sqrt{3}$	Simplified radical

Radicands may contain variables. When taking the square root of a variable, simply divide the exponent by the index, 2.

Example 1

$\sqrt{x^8}$	Divide the exponent by 2
$= x^4$	

Example 2

$$
\begin{aligned}
& -3\sqrt{18x^4y^6z^{10}} && \text{18 is divisible by 9, a perfect square} \\
=\ & -3\sqrt{9 \cdot 2x^4y^6z^{10}} && \text{Split into factors} \\
=\ & -3\sqrt{9} \cdot \sqrt{2x^4y^6z^{10}} && \text{Product Rule, take the square root of 9,} \\
& && \text{and divide the exponents by 2} \\
=\ & -3 \cdot 3x^2y^3z^5\sqrt{2} && \text{Multiply coefficients} \\
=\ & -9x^2y^3z^5\sqrt{2} && \text{Simplified radical}
\end{aligned}
$$

We can't always evenly divide the exponent on a variable by 2. Any whole number answer is how many of that variable will come out of the root. Any remainder is how many of the variable are left behind as part of the radicand.

Example 3

$$
\begin{aligned}
& \sqrt{20x^5y^9z^6} && \text{20 is divisible by 4, a perfect square} \\
=\ & \sqrt{4 \cdot 5x^5y^9z^6} && \text{Split into factors} \\
=\ & \sqrt{4} \cdot \sqrt{5x^5y^9z^6} && \text{Product Rule, take the square root of 4,} \\
& && \text{divide the exponents by 2} \\
=\ & 2x^2y^4z^3\sqrt{5xy} && \text{Simplified radical}
\end{aligned}
$$

Higher Roots

While square roots are the most common type of radical, we can take higher roots of radicands as well: cube roots, fourth roots, fifth roots, etc.

Examples

$\sqrt[3]{125} = 5$	$\sqrt[4]{-64} = -4$
$\sqrt[4]{81} = 3$	$\sqrt[7]{-128} = -2$
$\sqrt[5]{32} = 2$	$\sqrt[4]{-16}$ is undefined

Note: We can take an odd root of a negative number, because a negative number raised to an odd power is still negative.

Product Rule of Higher Roots

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Where *n* is a natural number,
and *a* and *b* are real numbers.

Example 1

$\sqrt[3]{54}$	54 is divisible by 27, a perfect cube
$= \sqrt[3]{27 \cdot 2}$	Split into factors
$= \sqrt[3]{27} \cdot \sqrt[3]{2}$	Product rule, take the cube root of 27
$= 3\sqrt[3]{2}$	Simplified radical

We can also take a higher root of a variable. Divide the exponent on the variable by the index. Any whole number answer is how many of that variable will come out of the root. Any remainder is how many of the variable are left behind as part of the radicand.

Example 2

$\sqrt[5]{x^{25}y^{17}z^3}$	Divide each exponent by 5 (*whole number outside, remainder inside*)
$= x^5y^3\sqrt[5]{y^2z^3}$	Simplified radical

Example 3

$2\sqrt[3]{40a^4b^8}$	Look for a perfect cube that is a factor of 40, the perfect cube is 8
$= 2\sqrt[3]{8 \cdot 5a^4b^8}$	Split the factors
$= 2\sqrt[3]{8} \cdot \sqrt[3]{5a^4b^8}$	Product Rule, take the cube root of 8, and divide the exponents by 3
$= 2 \cdot 2ab^2\sqrt[3]{5ab^2}$	Multiply coefficients
$= 4ab^2\sqrt[3]{5ab^2}$	Simplified radical