

Special Trigonometric Limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin Ax}{Bx} = \frac{B}{A}$$

$$\lim_{x \rightarrow 0} \frac{\sin Ax}{\sin Bx} = \frac{A}{B}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos Ax}{Ax} = 0$$

$$\lim_{x \rightarrow 0} \frac{\cos Ax - 1}{Ax} = 0$$

Even though the above limits are true, it is expected that you will be able to show your work when solving a trigonometric limit.

Examples

1. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

$$= \lim_{x \rightarrow 0} \left[\frac{\sin 3x}{x} \cdot \frac{3}{3} \right]$$

$$= 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x}$$

$$= 3(1)$$

$$= 3$$

- multiply both the numerator and denominator by 3

- rearrange the terms (**Reminder:** $\lim_{x \rightarrow a} cx = c \lim_{x \rightarrow a} x$)

- use the special limit $\lim_{x \rightarrow 0} \frac{\sin Ax}{Bx} = \frac{A}{B}$

2. $\lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x}$

$$= \lim_{x \rightarrow 0} \left[\frac{1 - \cos 7x}{x} \cdot \frac{7}{7} \right]$$

$$= 7 \lim_{x \rightarrow 0} \frac{1 - \cos 7x}{7x}$$

$$= 7(0)$$

$$= 0$$

- multiply both the numerator and denominator by 7

- rearrange the terms

- use the special limit $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

3. $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 4x}$

$$= \lim_{x \rightarrow 0} \left[\frac{\sin 5x \cdot \frac{5x}{5x}}{\sin 4x \cdot \frac{4x}{4x}} \right]$$

$$= \lim_{x \rightarrow 0} \left[\frac{\frac{\sin 5x}{5x} \cdot 5x}{\frac{\sin 4x}{4x} \cdot 4x} \right]$$

$$= \frac{5 \lim_{x \rightarrow 0} \frac{\sin 5x}{5x}}{4 \lim_{x \rightarrow 0} \frac{\sin 4x}{4x}}$$

$$= \frac{5}{4} \cdot \frac{1}{1}$$

$$= \frac{5}{4}$$

- multiply the numerator by $\frac{5x}{5x}$ and the denominator by $\frac{4x}{4x}$

- rearrange the terms

- cancel x s and apply the limit of a quotient property

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$