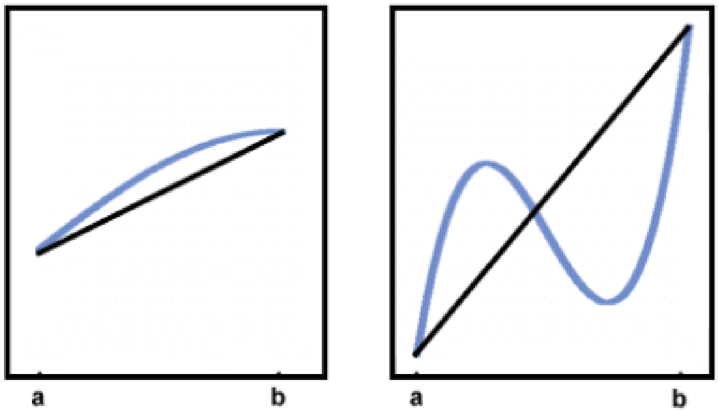
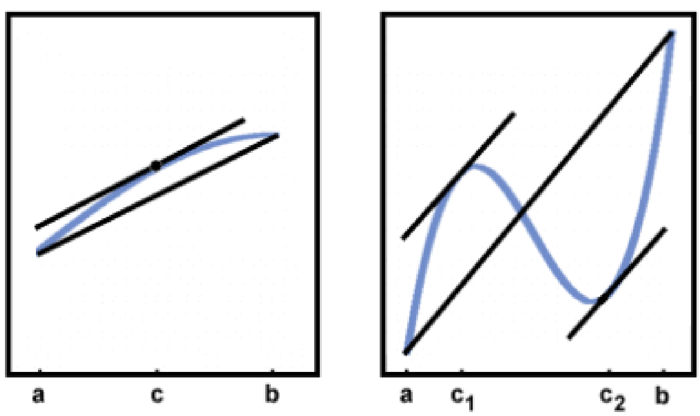
# The Mean Value Theorem

Connect any two points, $(a, f(a))$ and $(b, f(b))$, on a differentiable function $f$ to form a line:




Intuitively, it should be clear that we can find a point at $c$ between $a$ and $b$ where the tangent line is parallel to the secant drawn between $a$ and $b$. In other words, it should be possible to find a point such that the slope of the tangent at $c$ is the same as the slope of the secant line drawn from $a$ to $b$.




**The Mean Value Theorem** states that if $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then there exists a point at $c$ on $[a, b]$ for which the following is true:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

**Problem:** Use the Mean Value Theorem to find the *c* on [*a*, *b*] such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

**1:** $f(x) = x^2 - 4x$ on [2, 4]

$$f'(c) = \frac{0 - (-4)}{2} = 2$$

$$2c - 4 = 2$$

$$c = 3$$

**2:** $f(x) = \frac{1}{x}$ on [1, 2]

$$f'(c) = \frac{\frac{1}{2} - 1}{2 - 1}$$

$$= -\frac{1}{2}$$

$$-\frac{1}{c^2} = -\frac{1}{2}$$

$$c = \pm\sqrt{2}$$

## Rolle's Theorem

Rolle's Theorem is a special case of the Mean Value Theorem in which

$$f(a) = f(b).$$

If *f* is continuous on [a,b] and differentiable on (a, b), and $f(a) = f(b)$, then there is a point *c* on (*a*, *b*) where $f'(c) = 0$ .

The figure below should make clear that this is just a special case of the Mean Value Theorem: