The Six Trigonometric Ratios

The Three Primary Trigonometric Ratios

1. **Sine Ratio** $\sin\theta = \frac{opposite}{hypotenuse}$ $\sin\theta = \frac{y}{r}$

2. **Cosine Ratio** $\cos\theta = \frac{adjacent}{hypotenuse}$ $\cos\theta = \frac{x}{r}$

3. **Tangent Ratio** $\tan\theta = \frac{opposite}{adjacent}$ $\tan\theta = \frac{y}{x}$

The Three Reciprocal Trigonometric Ratios

1. **Cosecant Ratio**

$$\csc\theta = \frac{hypotenuse}{oppposite}$$

$$\csc\theta = \frac{r}{y}$$

The cosecant ratio is the reciprocal of the sine ratio.

$$\csc\theta = \frac{1}{\sin\theta}$$

2. **Secant Ratio**

$$\sec\theta = \frac{hypotenuse}{adjacent}$$

$$\sec\theta = \frac{r}{x}$$

The secant ratio is the reciprocal of the cosine ratio.

$$\sec\theta = \frac{1}{\cos\theta}$$

3. **Cotangent Ratio**

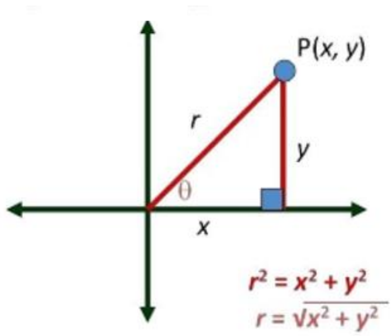
$$\cot\theta = \frac{adjacent}{opposite}$$

$$\cot\theta = \frac{x}{y}$$

The cotangent ratio is the reciprocal of the tangent ratio.

$$\cot\theta = \frac{1}{\tan\theta}$$

Suppose the angle θ is an angle in standard position. Given a point (x, y) on the terminal arm, at a distance r from the origin, we can determine the exact values of the six trigonometric ratios.

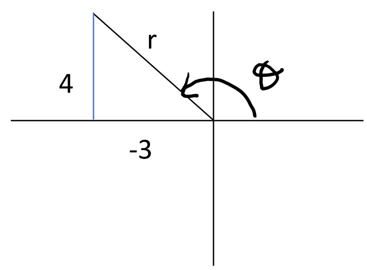


$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

Example 1

Given the point P(-3, 4), find the exact values of the six trigonometric ratios.



$r^2 = x^2 + y^2$

$r^2 = (-3)^2 + (4)^2$

$r^2 = 9 + 16$

$r^2 = 25$

$r = 5$ (**Note**: r must be positive, so we need only write the positive root)

$\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{-3}{5}$ $\tan\theta = \frac{4}{-3} = \frac{-4}{3}$

$\csc\theta = \frac{5}{4}$ $\sec\theta = \frac{5}{-3} = \frac{-5}{3}$ $\cot\theta = \frac{-3}{4}$

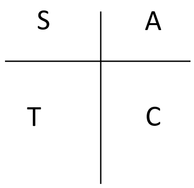
NOTE – If you are asked to find a trig ratio's exact value, leave the ratio as a fraction in simplest form. (No decimals!)
You may have to rationalize denominators so as not to leave radicals in the denominators of ratios.

Example 2

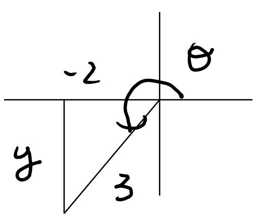
If $\sec\theta = -\frac{3}{2}$ and $\tan\theta > 0$, find the exact values of the other five trigonometric ratios.

$\sec\theta = \frac{r}{x}$ $\quad r = 3,\ x = -2$ (Remember, the radius must be positive.)

Use the CAST Rule to determine in which quadrant must the terminal arm lie?



Secant is the reciprocal of cosine, therefore it is negative in Quadrants 2 and 3.
Tangent is positive in Quadrants 1 and 3.
The terminal arm must lie in Quadrant 3.



Find y.

$$y^2 = r^2 - x^2$$

$$y^2 = (3)^2 - (-2)^2$$

$$y^2 = 9 - 4$$

$$y^2 = 5$$

$$y = -\sqrt{5}$$

You must choose the negative root.

$$\sin\theta = \frac{-\sqrt{5}}{3}$$

$$\cos\theta = \frac{-2}{3}$$

$$\tan\theta = \frac{-\sqrt{5}}{-2} = \frac{\sqrt{5}}{2}$$

$$\csc\theta = \frac{3}{-\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{-5} = -\frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{-2}{-\sqrt{5}} \cdot \frac{-\sqrt{5}}{-\sqrt{5}} = \frac{2\sqrt{5}}{5}$$