Pre-Calculus A 120

Worksheet – Applications of Exponential and Logarithmic Functions

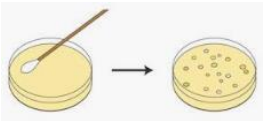
1. A sports car priced at \$60 000 depreciates at a rate of 14% per year. The value after t years is given by $A(t) = 60\,000(0.86)^t$ where A is the amount after depreciation. How long, to the nearest year, will it take for the value of the car to depreciate to \$18 000?

2. Welding is the most common way to permanently join metal parts together. Welders wear helmets fitted with a filter shade to protect their eyes from the intense light and radiation produced by a welding light.

   The filter shade number, N, is defined by the function $N = \frac{7(-\log T)}{3} + 1$, where T is the fraction of visible light that passes through the filter. Shade numbers range from 2 to 14, with a lens shade number 14 allowing the least amount of light to pass through.

   The correct filter shade depends on the type of welding. A shade number 12 is suggested for arc welding. What fraction of visible light is passed through the filter to the welder, as a percent to the nearest ten thousandth.

3. The number, N, of throat swab bacteria being grown in a culture after t hours, is given by the formula $N = N_0(10^{0.43t})$, where N_0 is the original number of bacteria. If there are initially 500 bacteria in the culture, determine how long it would take, to the nearest tenth of an hour, for the number of bacteria to grow to 1 million.



4. There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, F, of flower species that a butterfly feeds on and the number, B, of butterflies observed can be modelled by the function $F = -2.541 + 8.958 \log B$. Determine the number of butterfly observations in a region with 25 flower species.

5. The price of a famous brand name camera lens can be found by the equation $P = 14(1.1)^c$, where c is the circumference of the lens in centimeters and P is the price of the lens in dollars. Determine the diameter, the to the nearest tenth of a centimeter, of a camera lens which costs \$2500.

6. Earthquake intensity is measured by the Richter scale. The formula for the Richter rating of a given quake is given by $R = \log\left[\frac{I}{I_o}\right]$ where I_o is the threshold quake," or movement that can barely be detected, and the intensity I is given in terms of multiples of that threshold intensity.

   You have a seismograph set up at home, and see that there was an event while you were out that had an intensity of $I = 989I_o$. Given that a heavy truck rumbling by can cause a micro-quake with a Richter rating of 3 or 3.5, and "moderate" quakes have a Richter rating of 4 or more, what was likely the event that occurred while you were out?

7. A current, I_0, in amperes, falls to I amperes after t seconds according to the formula $I = I_0 e^{-kt}$. Determine the value of the constant, k, to the nearest whole number, if a current of 25 amperes falls to 2.5 amperes in 0.01 s.

8. The half-life of a medication is the amount of time for half of the drug to be eliminated from the body. The half-life of *Advil* or ibuprofen is represented by the equation, $R = M(0.5)^{\frac{t}{2}}$, where *R* is the amount of Advil remaining in the body, *M* is the initial dosage, and *t* is time in hours.

   a) A 200 milligram dosage of Advil is taken at 1:00 pm. How many milligrams of the medication will remain in the body at 6:00 pm?

   b) If a 200 milligram dosage of Advil is taken how many milligrams of the medication will remain in the body 12 hours later?

9. The logarithmic scale used to express the pH of a solution is $pH = -\log[H^+]$, where $[H^+]$ is the hydrogen ion concentration, in moles per litre (mol/L).

   Note: Solutions with pH value of less than 7 are acidic; solutions with a pH value of greater than 7 are basic; solutions with pH of 7 (such as pure water) are neutral.

   Suppose you test apple juice and find that the hydrogen ion concentration is $[H^+] = 0.0003$. Find the pH value and determine whether the juice is basic or acidic.

10. The equivalent amount of energy, E, in kilowatt-hours (kWh), released for an earthquake with a Richter magnitude of R is determined by the function $R = 0.67\log(0.36E) + 1.46$

   a) Describe how the function is transformed from $R = \log E$.
   b) The strongest earthquake in Eastern Canada occurred in 1963 at Charlevoix, Quebec. It has a Richter magnitude of 7.0. What was the equivalent amount of energy released, to the nearest kilowatt-hour?

Answer Key

1. 8 years

2. 0.0019%

3. 7.7 hours

4. 1187 butterfly observations

5. 17.3 cm

6. R = 3 (heavy truck rumbling by)

7. k = 230

8. 35.4 mg

9. a) 35.4 mg
   b) 3.1 mg

10. a) Vertically compressed by a factor of 0.67

    Horizontally expanded by a factor of 9.8

    Shifted 1.46 units up

    b) 5.16×10^8 kWh