Calculus 120

Worksheet – The Mean Value Theorem and Rolle's Theorem

The Mean Value Theorem (MVT)

If f is continuous on the closed interval [*a*, *b*] and differentiable on the open interval (*a*, *b*), then there exists a number *c* in (*a*, *b*) such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.

Determine whether the MVT can be applied to *f* on the closed interval. If the MVT can be applied, find all values of *c* given by the theorem. If the MVT cannot be applied, explain why not.

1. $f(x) = x^3$, $[0, 1]$

2. $f(x) = x^4 - 8x$, [0, 2]

3. $f(x) = \frac{x+1}{x}$, [-1, 2]

4. $f(x) = x^3 - x - 1$, [-1, 2]

Rolle's Theorem

Let *f* be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If $f(a) = f(b)$ then there is at least one number *c* in (a, b) such that $f'(c) = 0$.

Determine whether Rolle's Theorem can be applied to *f* on the closed interval. If Rolle's Theorem can be applied, find all values *c* in the open interval such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

1. $f(x) = x^2 - 5x + 4$, [1, 4]

2. $f(x) = x^{\frac{2}{3}} - 1$, [-8, 8]

3. $f(x) = \frac{x^2-2x-3}{x+2}$, [-1, 3]

4. $f(x) = \frac{x^2-1}{x}$, [-1, 1]

5. $f(x) = x\sqrt{6-x}$, [0, 6]

Answer Key

The Mean Value Theorem

1. $c = \frac{\sqrt{3}}{3}$
2. $c = \sqrt[3]{2}$
3. $f(x)$ is not continuous on [-1, 2] because there is an infinite discontinuity at $x = 0$. The MVT does not apply.
4. $c = 1$

Rolle's Theorem

1. $c = \frac{5}{2}$
2. The function is not differentiable at $x = 0$. Rolle's Theorem does not apply.
3. $c = -2 + \sqrt{5}$
4. The function is not continuous at $x = 0$. Rolle's Theorem does not apply.
5. $c = 4$

Answer Key

The Mean Value Theorem

1. $c = \frac{\sqrt{3}}{3}$
2. $c = \sqrt[3]{2}$
3. $f(x)$ is not continuous on [-1, 2] because there is an infinite discontinuity at $x = 0$. The MVT does not apply.
4. $c = 1$

Rolle's Theorem

1. $c = \frac{5}{2}$
2. The function is not differentiable at $x = 0$. Rolle's Theorem does not apply.
3. $c = -2 + \sqrt{5}$
4. The function is not continuous at $x = 0$. Rolle's Theorem does not apply.
5. $c = 4$