1.1 Part 1 Arithmetic Sequences

A sequence is a list of numbers that follow a logical pattern.

Ex. 3, 6, 9, 12, 15 + 3
     3, 6, 12, 24, 48 × 2
     3, 6, 10, 15, 21, 28 + 1 more each time
     3, 6, 9, 15, 24 add the previous 2 numbers

Arithmetic Sequences (based on addition)

Ex.1 7, 11, 15, 19, 23,… is an arithmetic sequence
     because 4 is added each time

We say \( t_1 = 7 \), \( t_2 = 11 \), \( t_3 = 15 \),

Determine the 20th term

\[
7 + 4(n-1)
\]

\[
= 7 + 76
\]

\[
= 83
\]

Determine the 200th term

\[
7 + 4(199)
\]

\[
= 7 + 796
\]

\[
= 803
\]

Ex.3 Given the arithmetic sequence 47, 60, 73, 86,
determine \( t_{64} \)

\[
t_{64} = 47 + 13(63) = 866
\]

In general, given an arithmetic sequence \( t_1, t_2, t_3, \ldots, t_n \)

\[
t_n = a + d(n-1)
\]

\( a \) = first term
\( d \) = common difference
\( n \) = number of terms
\( t_n \) = general term or \( n^{th} \) term

Ex.4 Given the a.s. 24, 41, 58, 75, determine \( a \), \( d \), and the next few terms

\( a = 24 \)
\( d = 41 - 24 = 17 \)

\( t_5 = 92 \)
\( t_6 = 109 \)
\( t_7 = 126 \)

Ex.5 Given the a.s. -8, 5, 18, 31, determine

\( a = -8 \)

\( b) \ t_{31} \)

\[
t_{31} = -8 + 13(31 - 1)
\]

\[
= -8 + 390
\]

\[
= 382
\]

\( c) \ t_n \)

\[
t_n = -8 + 13(n - 1)
\]

\[
= -8 + 13n - 13
\]

\[
= 13n - 21 \leftarrow \text{general term} 
\]
Ex. 6 a) Given the a.s. 7, 12, 17, ..., determine the general term

\[ t_n = 7 + 5(n - 1) \]
\[ = 7 + 5n - 5 \]
\[ = 5n + 2 \]

b) Use the general term to determine.

i) \( t_7 \)

\[ t_7 = 5(7) + 2 \]
\[ = 37 \]

ii) \( t_{23} \)

\[ t_{23} = 5(23) + 2 \]
\[ = 117 \]

Ex. 7 For the following sequences, determine the missing terms

a) \(-11, -8, -7, 16, 25\) \(d = 25 - 16 = 9\) \(\therefore -11, -8, -7\)

b) \(2, 8, 10, 12, 14\)
\[8 + 3d = 14\]
\[3d = 6\]
\[d = 2\]
\(\therefore 6, 8, 10, 12, 14\)

c) \(12, 14.5, 17, 19.5, 22\)
\[12 + 4d = 22\]
\[4d = 10\]
\[d = 2.5\]
\(\therefore 12, 14.5, 17, 19.5, 22\)

d) \(t_4 = 7, t_6 = 16\)
\[-6.5, -2, 2.5, 7, 11.5, 16\]
\[7 = 2d = 16\]
\[2d = 9\]
\[d = 4.5\]
1.1 Part 2 Arithmetic Sequences

Ex.1 Determine the general term for 11.8, 13, 14.2, ...

\[ t_n = 11.8 + 1.2(n-1) \]
\[ = 11.8 + 1.2n - 1.2 \]
\[ t_n = 1.2n + 10.6 \]

Ex.2 In the a.s. 8, 13, 18, ..., which term is 333?

\[ a = 8 \]
\[ d = 5 \]
\[ t_n = 333 \]
\[ n = ? \]

\[ 333 = 8 + 5(n-1) \]
\[ 325 = 5n - 5 \]
\[ 330 = 5n \]
\[ n = 66 \]

Ex.3 How many terms are in the a.s 34, 25, 16, ..., -245?

\[ a = 34 \]
\[ d = -9 \]
\[ t_n = -245 \]
\[ n = ? \]

\[ -245 = 34 - 9(n-1) \]
\[ -279 = -9n + 9 \]
\[ -288 = -9n \]
\[ n = 32 \]

Ex.4 Determine the first term if \( t_2 = 348 \) and \( d = 11 \).

\[ 348 = a + 11(2-1) \]
\[ 348 = a + 220 \]
\[ a = 128 \]

Ex.5 Given the length of each side of the small triangles is 1, determine the following.

\[ \begin{array}{ccc}
\Delta & \Delta & \Delta \\
\downarrow & \downarrow & \downarrow \\
t_1 & t_2 & t_3 \\
\end{array} \]

a) Create a chart representing the term number and the perimeter

<table>
<thead>
<tr>
<th>term</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>perimeter</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>...</td>
</tr>
</tbody>
</table>

b) Determine the general term for part a)

\[ t_n = 3 + 3(n-1) \]
\[ = 3 + 3n - 3 \]
c) Use the general term to determine the perimeter of the 10th diagram

\[ t_n = 3 + 3(12-1) \]

\[ t_{12} = 3(12) = 36 \]

d) Which figure has a perimeter of 108?

\[ 108 = 3n \]

\[ n = 36 \]

Ex 6 A person runs 4 km on day 1 and increases the distance ran by the same amount each day. By day 5, the person runs 7 km.

a) Determine the general term

\[ a = 4, \ldots, 7 \]

\[ 4 + 4d = 7 \]

\[ 4d = 3 \]

\[ d = 0.75 \]

\[ t_n = 4 + 0.75(n-1) \]

\[ t_5 = 4 + 0.75(5-1) = 7 \]

b) Use the general term to determine on which day the person will run 19 km.

\[ 19 = 0.75n + 3.25 \]

\[ 15.75 = 0.75n \]

\[ n = 21 \]

\[ \therefore \text{day 21} \]
1.2 Arithmetic Series

Sequence: 3, 7, 11, 15, 19, ...
Series: 3 + 7 + 11 + 15 + 19 + ...

Add: 1 + 2 + 3 + ... + 98 + 99 + 100

Ex. 1 Determine the sum of the first 20 terms of 7 + 11 + 15 + ...

\[ t_{20} = 7 + 4(20-1) \]
\[ = 7 + 76 \]
\[ = 83 \]
\[ 90 \]
\[ \therefore S_{20} = 7 + 11 + 15 + ... + 75 + 79 + 83 \]
\[ = 90 \times 10 = 900 \]

In general \[ S_n = \frac{n}{2} (a + t_n) \]

Ex. 2 Determine \( S_{30} \) for 17 + 11 + 5 + ...

\[ a = 17 \]
\[ t_{30} = 17 - 6(30-1) \]
\[ n = 30 \]
\[ d = -6 \]
\[ t_n = ? \]

\[ S_{30} = \frac{30}{2} (17 - 157) \]
\[ = -15 \]
\[ = 15 (-140) \]
\[ = -2100 \]

We can also use the formula \[ S_n = \frac{n}{2} (2a + d(n-1)) \]

\[ S_n = \frac{30}{2} (2(17) - 6(30-1)) \]

Use if we know \( a, n, \) and \( d \)

Ex. 3 Determine the sum of 7 + 14 + 21 + ... + 266

\[ a = 7 \]
\[ d = 7 \]
\[ n = ? \]
\[ t_n = 266 \]
\[ 266 = 7 + 7(n-1) \]
\[ 259 = 7(n-1) \]
\[ n = 38 \]

\[ S_{38} = \frac{38}{2} (7 + 266) \]

\[ = 5187 \]
Ex. 4 If \( a = -6 \), \( t_n = 21 \), and \( S_n = 75 \), determine \( n \).

\[
S_n = \frac{n}{2} (a + t_n)
\]

\[
75 = \frac{n}{2} (-6 + 21)
\]

\[
150 = 15n
\]

\[
n = 10
\]

Ex. 5 The 2nd term of an arithmetic series is 17 and the 6th term is 29. Determine the sum of the first 13 terms.

\[
+ 17 + \ldots + 29
\]

\[
a = 14 \quad S_n = \frac{13}{2} (2(14) + 3(13-1))
\]

\[
d = 3 \quad = 6.5 (28 + 36)
\]

\[
n = 13 \quad = 416
\]

\[
d = 3
\]

\[
\therefore 14 + 17 + 20 + 23 + 26 + 29 + \ldots
\]

Ex. 6 A pile of bricks is arranged in rows. The number of bricks in each row form the a.s. 65, 59, 53, ... How many bricks are there in the first 9 rows?

\[
a = 65
\]

\[
d = -6
\]

\[
S_n = \frac{a}{2} (2(65) - 6(9-1))
\]

\[
n = 9
\]

\[
= 4.5 (130 - 48)
\]

\[
= 4.5 \times 82
\]

\[
= 369 \text{ bricks.}
\]