Two-Dimensional Motion

Objects don't always move in a straight line. When an object moves in two dimensions, we must look at vector components.

The most common kind of two-dimensional motion you will encounter is projectile motion.
Projectiles

A projectile is any object which once set into motion continues in motion by its own inertia and is influenced only by the downwards force of gravity.

There are a variety of examples of projectiles. In all cases the influence of air resistance must be negligible.

- an object dropped from rest
- an object thrown vertically upwards
- an object thrown upwards at an angle
- an object launched with an initial horizontal velocity from an elevated position

Since there is only one force acting on a projectile, gravity, all projectiles have the same free-body diagram:
The Path of a Projectile

First Law of Motion

Without gravity, an object in motion will continue in motion with the same speed and in the same direction.

Second Law of Motion

If an object is influenced by an unbalanced force, it will accelerate in the direction of that force.

Therefore, if an object is influenced by gravity, it will accelerate downwards and fall below its inertial path and follow a *parabolic trajectory* (path).
Drop versus Launch

At the same moment that the ball on the left is dropped, the ball on the right is launched horizontally.

Since perpendicular components of motion are independent of each other, gravity does not affect a projectile's horizontal motion.
Check for Understanding

**Projectile Motion**

1. Consider these diagrams in answering the following question.

   A

   B

   C

   D

   E

   Which diagram (if any) might represent:

   a) the initial horizontal velocity?
   b) the initial vertical velocity?
   c) the horizontal acceleration?
   d) the vertical acceleration?
   e) the net force?

2. Supposing a snowmobile is equipped with a flare launcher which is capable of launching a sphere vertically (relative to the snowmobile). If the snowmobile is in motion and launches the flare and maintains a constant horizontal velocity after the launch, then where will the flare land (neglect air resistance).

   **Simulation**
   The Truck and The Ball

   Imagine a pickup truck moving with a constant speed along a city street. In the course of its motion, a ball is projected straight upwards by a launcher located in the bed of the truck. Imagine as well that the ball does not encounter a significant amount of air resistance. What will be the path of the ball and where will it be located with respect to the pickup truck? How can the motion of the ball be described? And where will the ball land with respect to the truck?

3. The trajectory (path) of a projectile is ________________________.
The Truck and The Ball

Summary

- The ball follows a parabolic path.
- As the ball rises towards its peak, it slows down because it undergoes a downward acceleration.
- As the ball falls toward the ground, it speeds up because it undergoes a downward acceleration.
- The time it takes the ball to reach its peak is the same amount of time it will take to return to its initial height.
- The ball is always located directly above the launcher from which it was projected.
- We must assume the influence of air resistance is negligible.
4. The diagram at the right depicts the trajectory of a projectile; the position of the projectile after each consecutive second is shown. The arrows depict the horizontal and vertical components of the velocity. Use your understanding of projectiles to fill in the blanks.

\[
\begin{align*}
&\text{t}=0 \text{ sec}, \quad V_y^0=0 \text{ m/s} \quad \leftarrow V_x^0 = 20 \text{ m/s} \\
&\text{t}=1 \text{ sec}, \quad V_y^1 \quad \downarrow V_x^1 \\
&\text{t}=2 \text{ sec}, \quad V_y^2 \quad \downarrow V_x^2 \\
&\text{t}=3 \text{ sec}, \quad V_y^3 \quad \downarrow V_x^3 \\
&\text{t}=4 \text{ sec}, \quad V_y^4 \\
\end{align*}
\]

5. Anna Litical drops a ball from rest from the top of an 80 m high cliff. How much time will it take for the ball to reach the ground.

6. A cannon ball is launched horizontally from the top of an 80 m high cliff. How much time will it take for the ball to reach the ground.
Calculations

Gravity only affects the vertical motion of the balls (both balls accelerate downwards). We will have to use the kinematic equations to determine a projectile's vertical velocity and displacement.

No forces affect the projectile's horizontal motion. We will have to use \( v = \frac{d}{t} \) for the projectile's horizontal velocity and displacement.

Both motions take place during the same time interval. The time interval links the two motions.

The horizontal distance the projectile travels is called its range.

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**Projectile Charts**

$\begin{array}{|c|}
\hline
\text{Horizontal Motion} & \text{Vertical Motion} \\
\hline
v_x & 0 \text{ m/s} \\
\hline
d_x & v_{yi} \\
\hline
a_x & 0 \text{ m/s}^2 \\
\hline
t & v_{yf} \\
\hline
\end{array}$

$\begin{array}{|c|}
\hline
\hline
\end{array}$

$\begin{array}{|c|}
\hline
\text{Vertical Motion} \\
\hline
v_{yi} & 0 \text{ m/s} \\
\hline
v_{yf} & \\
\hline
d_y & \\
\hline
a_y & -9.8 \text{ m/s}^2 \\
\hline
t & \\
\hline
\end{array}$
Sample Problem

A soccer ball is kicked horizontally off a 22.0 m high hill and lands a distance of 35.0 m from the edge of the hill. Determine the initial horizontal velocity of the soccer ball.

<table>
<thead>
<tr>
<th>Horizontal Motion</th>
<th>Vertical Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_x$</td>
<td>$v_{yi}$ 0 m/s</td>
</tr>
<tr>
<td>$d_x$</td>
<td></td>
</tr>
<tr>
<td>$a_x$ 0 m/s²</td>
<td>$v_{yf}$</td>
</tr>
<tr>
<td>$t$</td>
<td>$d_y$</td>
</tr>
<tr>
<td></td>
<td>$a_y$ -9.8 m/s²</td>
</tr>
<tr>
<td></td>
<td>$t$</td>
</tr>
</tbody>
</table>
A pool ball leaves a 0.60 m high table with an initial horizontal velocity of 2.4 m/s. What is the pool ball's velocity just before it hits the ground?

### Sample Problem

#### Horizontal Motion

<table>
<thead>
<tr>
<th>$v_x$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_x$</td>
<td></td>
</tr>
<tr>
<td>$a_x$</td>
<td>0 m/s²</td>
</tr>
<tr>
<td>$t$</td>
<td></td>
</tr>
</tbody>
</table>

#### Vertical Motion

| $v_{y_i}$ | 0 m/s |
| $v_{y_f}$ |        |
| $d_y$     |        |
| $a_y$     | -9.8 m/s² |
| $t$       |        |
Sample Problem

An airplane is dropping supplies to northern villages that are isolated by severe blizzards and cannot be reached by land vehicles. The airplane is flying at an altitude of 785 m and at a constant horizontal velocity of 33.5 m/s. At what horizontal distance before the drop point should the co-pilot drop the supplies so that they will land at the drop point? (Neglect air friction.)

\[
\begin{align*}
\text{Horizontal Motion} & \\
\text{Vertical Motion} & \\
\begin{array}{|c|c|}
\hline
v_x & 0 \text{ m/s} \\
d_x & \\
a_x & 0 \text{ m/s}^2 \\
t & \\
\hline
\end{array}
\end{align*}
\]
Propel Bottle Project
Projectiles Launched at an Angle

If a projectile is launched at an angle to the horizontal, then the initial velocity of the projectile has both a horizontal and vertical component.

Reminder

*Vector resolution* is the process of taking a single vector at an angle and separating it into two perpendicular components.

\[ v_{xi} = v \cos \theta \quad v_{yi} = v \sin \theta \]

The time for a projectile to rise to its maximum height can be found by using the kinematic equation:

\[ a = \frac{v_{yf} - v_{yi}}{\Delta t} \]

For a projectile which lands at the same height at which it started, the total time of flight is twice the time to rise to the peak.

*The trajectory of a projectile is symmetrical about its maximum height.*
The initial launching velocity, $v$, must be *resolved* into vertical and horizontal components, $v_{yi}$ and $v_{xi}$.

If it takes a projectile 4.0 s to rise to its peak, then it will take a total of 8.0 s to move through the air from start to finish.

$$t_{\text{total}} = 2(t_{\text{up}})$$
http://www.youtube.com/watch?v=JNYSeIBA5Gk&feature=related
Sample Problems

A ball is shot at a speed of 25.0 m/s from a cannon at an angle of 30.0° above the horizontal. What is the ball's range?

Horizontal Motion

\[
\begin{array}{c|c}
  v_x & v_{yi} \\
  \hline
d_x & v_{yi} \\
  \hline
a_x & v_{yf} \\
  0 \text{ m/s}^2 & \\
  \hline
t & d_y \\
  & a_y \\
  & -9.8 \text{ m/s}^2 \\
  & t
\end{array}
\]

Vertical Motion
At the 1968 Olympic Games in Mexico City, Bob Beamon set a new world record in the long jump. His initial velocity was 9.435 m/s at an angle of 40.0° to the horizontal.

a) What was his maximum height?
b) What was his range?

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<td>$v_{yf}$</td>
</tr>
<tr>
<td>$a_x$</td>
<td>$d_y$</td>
</tr>
<tr>
<td>0 m/s²</td>
<td>$a_y$</td>
</tr>
<tr>
<td>$t$</td>
<td>-9.8 m/s²</td>
</tr>
<tr>
<td>$t$</td>
<td>$t$</td>
</tr>
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</table>
Video Clip
Men's Long Jump,
Tokyo 1991
A circus stunt person was launched as a human cannon ball over a Ferris wheel. His initial velocity was 24.8 m/s at an angle of 55°. (Neglect friction)

(a) Where should the safety net be positioned?

(b) If the Ferris wheel was placed halfway between the launch position and the safety net, what is the maximum height of the Ferris wheel over which the stunt person could travel?

(c) How much time did the stunt person spend in the air?

**Horizontal Motion**

| $v_x$ |  
| $d_x$ |  
| $a_x$ | $0 \text{ m/s}^2$ |

| $t$ |  

**Vertical Motion**

| $v_{yi}$ |  
| $v_{yr}$ |  
| $d_y$ |  
| $a_y$ | $-9.8 \text{ m/s}^2$ |

| $t$ |  